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Probabilistic Estimation of Recoverable Reserves

by

Chad Travis Neufeld

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Master of Science**.

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Probabilistic Estimation of Recoverable Reserves** submitted by Chad Travis Neufeld in partial fulfillment of the requirements for the degree of **Master of Science** in *Mining Engineering*.

Dr. Clayton V. Deutsch (Supervisor)

Dr. Oy Leuangthong (Co-supervisor)

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To Laura. Your love and support mean the world to me.

To my parents Craig and Donna.

Abstract

Reserve estimates are critical to mining projects. Mine productivity and equipment selectivity must be balanced and considered in the estimation of recoverable reserves. This has been a long-standing challenge in mining geostatistics since the early days of the discipline. Many techniques have been developed and others have evolved to address this problem.

This thesis reviews conventional and emerging approaches to resource/reserve estimation. Specifically, change of support models, kriging, uniform conditioning and simulation are considered. These different approaches are then illustrated using data from a gold deposit, and compared on the basis of their grade tonnage curves. This comparison revealed that the indirect lognormal, discrete Gaussian model, uniform conditioning with kriging and simulation yielded reasonable and comparative results. The use of kriging and the affine correction performed poorly relative to the other methods. Guidelines for consideration and implementation of these methods are provided.

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$\bullet$	point support volume
v	block support volume, often referred to as the SMU volume
V	larger block support volume, often referred to as the panel volume
A	deposit support volume
$\mathbf{u}$	location in space
$\mathbf{h}$	distance vector
Z_v	random variable for support size v
Y_v	transformed random variable for support size v
$v(\mathbf{u})$	domain of volume v centered at $\mathbf{u}$
$V(\mathbf{u})$	domain of volume V centered at $\mathbf{u}$
$z_v(\mathbf{u})$	grade for a block of size v at $\mathbf{u}$
$y_v(\mathbf{u})$	transformed grade for a block of size v at $\mathbf{u}$
$t_v(\mathbf{u})$	tonnage for a block of size v at $\mathbf{u}$
m	mean value
$E\{Z\}$	expected value, or mean, of Z
σ^2	variance
$Var\{Z\}$	variance of Z
CV	coefficient of variation (σ/m)
$f(z)$	probability density function of Z
$F(z)$	cumulative distribution function of Z
$g(y)$	probability density function of a standard Gaussian distribution
$G(y)$	cumulative distribution function of a standard Gaussian distribution
q	quantile of a distribution
$\gamma(\mathbf{h})$	semi-variogram function at lag $\mathbf{h}$
$C(\mathbf{h})$	covariance function at lag $\mathbf{h}$
$\overline{\gamma}(V,v)$	average variogram value between the domains V and v
$D^2(v,V)$	dispersion variance of the smaller blocks v within the larger blocks V

f	variance reduction factor
$\Phi(y)$	Gaussian anamorphosis function
ϕ_n	coefficient of order n of the Gaussian anamorphosis
$H_n(y)$	Hermite polynomial of order n for a Gaussian value y
r	change of support coefficient, used in uniform conditioning
λ_i	weight assigned to sample i
$z^*(\mathbf{u})$	estimate of Z at $\mathbf{u}$
$y^*(\mathbf{u})$	estimate of Y at $\mathbf{u}$
$y_S(\mathbf{u})$	simulated value at $\mathbf{u}$
$R(\mathbf{u})$	random residual at $\mathbf{u}$
σ_E^2	estimation variance
z_c	cutoff grade
p	metal price
c_o	ore mining cost
c_t	ore processing cost
c_w	waste mining cost
rec	processing recovery factor
T_0	total tonnage contained in the deposit
$T(z_c)$	tonnage above cutoff
$Q(z_c)$	quantity of metal above cutoff
$M(z_c)$	average grade above cutoff
$P(z_c)$	proportion of tonnes above cutoff
SMU	selective mining unit

Chapter 1

Introduction

Recoverable reserves/resources are integral to all mineral projects. Different types of estimates are required at different stages of the mine life: global reserves are needed at the project development stage, while local reserves are required for detailed mine planning and pit optimization. The estimation of reserves requires economic evaluation and detailed open pit/underground planning. The estimation of reserves is made more complex when one considers issues related to equipment and the corresponding scale of mining production.

There are three types of block models, or estimates, that can be considered for any mining operation (Paraphrased from [7]). Each model type has a different goal and different resource/reserve predictions:

Type 1 (Best Estimates): Models whose block grade estimates are constructed to be the best estimate at a location using the data available are classified as type 1 models. The main consideration when constructing this type of model is to best predict the unknown value at a location where there is no sample. Kriging was developed to provide this capability. These models do not necessarily provide good estimates of the global resources or reserves because they are often too smooth and do not anticipate the additional information that will be available at the time of mining. This motivates us to consider alternative models.

Type 2 (Tuned for Reserves): Models whose block grade estimates are used to predict the tonnes and average grade of ore material that will be recovered over the life-of-mine are classified as Type 2. Typically, type 2 models are constructed at the initial mine planning stages from relatively sparse exploration drilling. The estimates are used for long term mine planning, and pit optimization. The estimates are made with the knowledge that they will not be used for final selection at the time of mining. For this reason, the location of the high and low grade estimates is not essential. It is important that type 2 estimates make an accurate prediction of the recoverable reserves at the time of mining. The estimation parameters are tuned to give unbiased recoverable reserve estimates given the available information and any information on future grade control and mining practices.

Type 3 (Decision Making): Models whose block grade estimates are used for selection at the time of mining are classified as Type 3. Individual block volumes

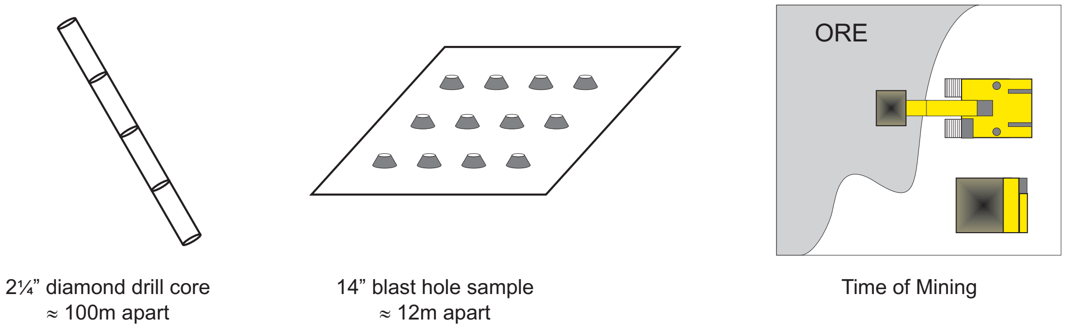

Figure 1.1: Stages of reserve estimation.

correspond to a selective mining unit (SMU). The estimates at this stage are usually associated with the mining equipment and some assessment of the consequences associated with under and over estimation. The grade of each block typically estimated from neighboring blast hole (BH) grades or dedicated grade control drill samples. The use of these estimates and subsequent hand smoothing to distinguish between ore and waste is commonly known as grade control.

Consider Figure 1.1. At the initial stages of mine exploration the only data available is sparsely sampled exploration holes; typically core from diamond drilling or cuttings from reverse circulation drilling. At the time of grade control, additional samples will be available. These are taken from blast hole cuttings or from dedicated grade control drilling. Grade control estimates are used to decide where the actual mining will take place.

The idea of type 2 estimates is to use the sparsely sampled exploration holes to predict the material that the shovel or loader will actually mine. The estimates need to account for the selectivity of the mining equipment, the future information available from the grade control samples, and any errors that may occur. The challenge with these estimates is that they are often locally biased, that is, high values are too high and low values are too low. This happens because the estimates are compensating for future information that is not available at the present time.

The notion of a selective mining unit, or SMU, is confusing. Many people associate an SMU with the mining equipment. This is correct at the time of mining; however, during the reserve estimation, the SMU does not coincide with the mining equipment, it has to be correctly sized to accurately predict the results of the mining. The SMU has to account for the mining equipment, the imperfect information at the time of mining and dilution. The SMU size will change from initial mine exploration and planning to the mine development and mining stages.

Recoverable reserves are a function of the selectivity that can be achieved by the equipment being used. Large equipment is more productive than small equipment, but this comes at the price of selectivity. Larger mining equipment cannot distinguish ore and waste as easily or accurately as small mining equipment. The selective mining unit (SMU) could be defined as *the smallest volume of material on which ore waste classification is determined* [19]; hence it is a clear measure of the equipment selectivity and has a large impact on the reserve calculation (see [10] for more information). As mentioned above, the SMU also accounts for imperfect selection due to incomplete information.

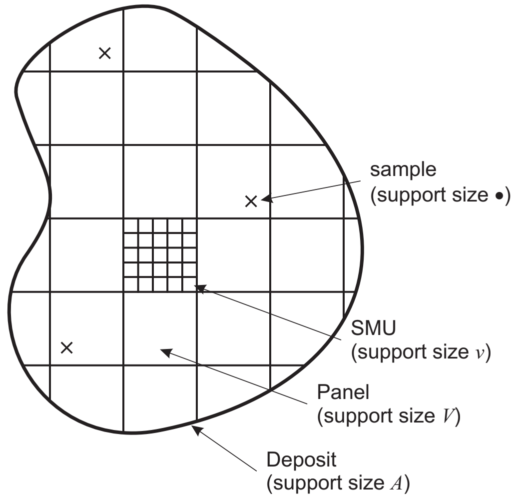

Figure 1.2: Typical setting for reserve calculation: samples are represented by the X's; the SMU's denote the small blocks, v; the panels as represented by the larger blocks, V; and the deposit is considered as the entire area, A.

While SMU's are considered the smallest volumes for production, reserves are calculated for larger mine planning blocks, typically referred to as panels. Calculation of panel grades, tonnages, and ore/waste classification are based on the constituent SMU's within the panel. The panel reserves can be further upscaled to provide an estimate of the reserves for the entire deposit. Figure 1.2 shows a schematic illustration of the relation between these scales of production, planning and reporting.

Achieving the balance between productivity and selectivity is a challenge, yet the effect of dilution due to scale of production presents another issue for consideration. Consider assays from an exploration drilling program. There will typically be samples with very low grades and samples with very high grades; these extreme values contribute to a histogram with high variance. If the samples are composited to a larger scale, higher grade material will be averaged with lower grade material. The effect of this averaging leads to a histogram of composited data that has a lower variance than the assays (see Figure 1.3). Clearly, the distribution of grades within the deposit changes as the size of the sample, or block, increases.

Given these complex, inter-related issues, it is easy to see why so many different resource estimation methods have been developed for different deposit types and settings. Incorporating scale effects in the resource estimate is very important. There are analytical change of support models to correct the point scale histogram to one that approximates the SMU scale histogram. Global resources can then be calculated directly from the upscaled histogram. Local resources can be estimated through linear, non-linear, and simulation based algorithms. The local resources can be upscaled to get global resources.

This thesis will compare some of the more common reserve estimation techniques on the basis of (i) applicability, (ii) amount of time and effort required for implementation, and (iii) accuracy of the results. A background review of volume variance, resource and reserve calculation, and the information effect is first presented. This

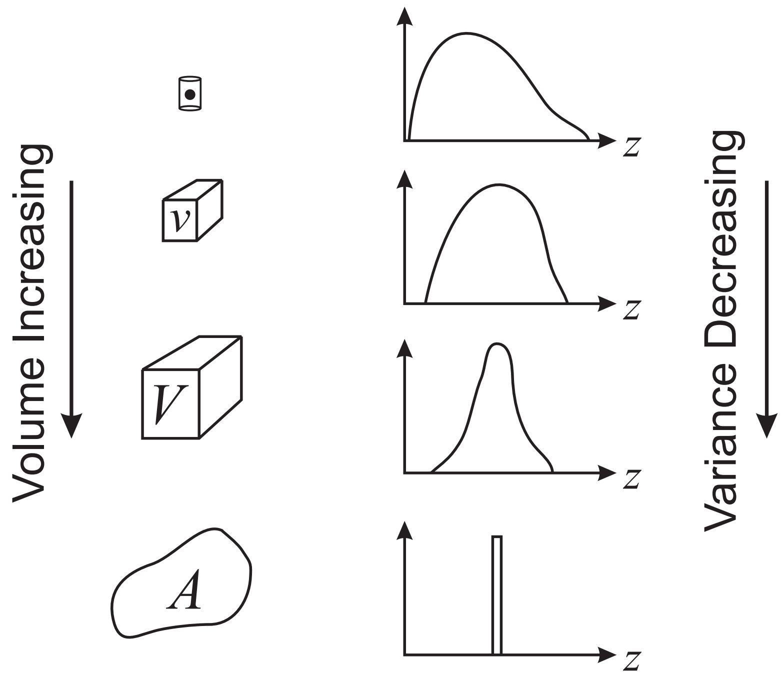

Figure 1.3: Change in the histogram with a change in the support size (redrawn from Deutsch, 2001).

is followed by a distinction between global and local resource estimation methods. A comparative example is included to demonstrate the differences between the methods. A contrived example illustrates the calculation of an effective SMU size that can be used for reserve estimation by incorporating the information effect.

1.1 Background

The scale issue remains a long standing challenge in geostatistics; certainly the mining industry has struggled with its complexity since the pioneering work of Krige and Sichel in the 1950s [11, 18]. Since that time, many researchers have worked to quantify the affect of volume on the spatial variability of the phenomena, and how this affects the variability in the grades distribution. A brief summary of average variogram (gammabar) values, dispersion variances, and how they relate to block size is given.

Another problem that must be considered is dilution. Dilution is important for two reasons: (1) the mining equipment, or method, cannot mine at a scale small enough to select between ore and waste; Free Selection, and (2) there will always be errors in classifying material as ore and waste due to sampling errors and incomplete sampling; Perfect Selection. The reserve calculation should account for dilution, not being able to freely select ore and waste, and misclassified blocks, not being able to perfectly classify each block. The impact that additional sample information has is called the information effect.

Recoverable reserves can be calculated from a histogram of grades, or from an estimated block model. The methodology and notation are presented.

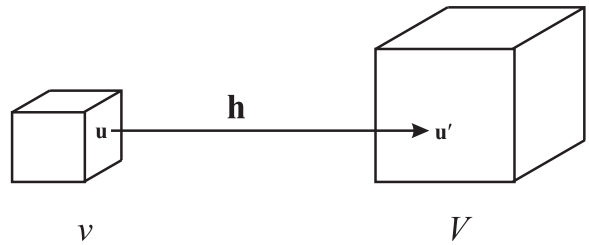

Figure 1.4: Schematic illustration of two arbitrary volumes, v and V, separated by a lag distance, $\mathbf{h}$.

1.1.1 Scale Effect

Gammabar. The average variogram, or gammabar, $\overline{\gamma}(V, v)$, is a widely used measure to estimate the variability between two arbitrary volumes. Gammabar values represent the mean value of $\gamma(\mathbf{u})$ when the head of the variogram describes the block $V(\mathbf{u})$ and the tail describes the block $v(\mathbf{u'})$ where $\mathbf{u}$ and $\mathbf{u'}$ are location vectors (see Figure 1.4). The blocks v and V may or may not be the same size, they may be separated by some distance, and/or they may be irregularly shaped.

Gammabar values can be calculated exactly with the following integral:

$$\overline{\gamma}(V, v) = \frac{1}{V \cdot v} \int_{V(\mathbf{u})} dx \int_{v(\mathbf{u'})} \gamma(x - x')\, dx' \qquad (1.1)$$

Alternatively, the gammabar can be numerically approximated by a sextuple summation (3 directions for each volume) taken over the discretization of the two volumes. This is simple, relatively fast and works for any variogram type and size/shape of the volumes. This flexibility makes the numerical approach the common modern approach.

Consider the case of calculating the gammabar of a volume with itself. If the volume v is very small, say a point support, the $\overline{\gamma}(v, v)$ will be equal to zero since the separation vector, $\mathbf{h}$, will equal zero. In contrast, a much larger volume, say the domain will involve averaging the variogram of points separated by very large distances, $\mathbf{h}$; this results in a $\overline{\gamma}(A, A)$ nearly equal to the variance of the field. Thus, as the volume increases, the gammabar values will also increase and tend towards the variance of the field.

Dispersion Variance. The dispersion variance, $D^2(v, V)$, is a measure of the variability of the small volume v within a larger volume V. Dispersion variances can be estimated in one of two ways: (1) use the available data and estimate the block variance by upscaling the data directly, and (2) estimate the block variance using the fitted variogram model. Ideally, both methods would be used and any differences reconciled. However, there is usually insufficient data to perform method 1. As a result, method 2 has become the accepted default practice for estimating block variances.

The dispersion variance can be written as:

$$D^2(v, V) \quad = \quad E\left[(z_v - m_V)^2\right]$$

where z_v is the are the sample points with a volume v contained within the larger volume V, and m_V is the mean of the larger volume V. This requires sufficient data for calculating the dispersion variance. The dispersion variance can be rewritten in terms of the variogram [9]:

$$D^2(v,V) = \bar{\gamma}(V,V) - \bar{\gamma}(v,v) \tag{1.2}$$

Equation (1.2) allows the easy calculation of a dispersion variance given a variogram model and a volume. Any dispersion variance estimated with the variogram is sensitive to the nugget effect.

Dispersion variances are additive. The additivity of variance property was used to establish Krige's relation [2, 9]:

$$D^2(v,A) = D^2(v,V) + D^2(V,A) \tag{1.3}$$

Therefore, the variance of the small blocks v in the area A is equal to the variance of the small blocks v within the big blocks V plus the variance of the big blocks V in the area A, that is $v \subset V \subset A$. This equation can be rearranged to solve for any of the terms.

1.1.2 Perfect and Free Selection

It is impossible for the mining equipment to perfectly mine all the ore as ore and all the waste as waste. The notion of perfect and free selection is important. Consider the ore/waste map shown in Figure 1.5. Ore is shown as dark gray and waste is shown as light gray. The dark black line is a proposed ore/waste boundary for the mining equipment. Perfect selection assumes that each block could be perfectly mined as a cube. Free selection assumes that each ore block can be mined as ore, and that each waste block can be mined as waste. These are not realistic assumptions. The few waste blocks contained within the ore will likely be mined as ore, and the orphaned ore blocks will likely be mined as waste.

1.1.3 Information Effect

Ore and waste must be classified for mining. Classification is done using blast hole or grade control samples. The samples are collected near the time of mining and are used to estimate the block grades. Each block is classified according to its estimated grade. The estimated grades will never exactly match the true block grades. The amount of additional information that the grade control samples bring is called the information effect.

The information effect quantifies the affect that the grade control samples have on the recovered reserves. Widely spaced samples will not predict the mineral grades as accurately as closely spaced samples. The result is misclassified blocks and a reduction in the recovered mineral grade. The information effect can be accounted for in a reserve estimate.

1.1.4 Recoverable Reserve Calculation

A change in volume affects the spatial variability of the grades. A change in the grades distribution must necessarily affect the recoverable reserves calculation. This section reviews the different approaches for reserves calculation.

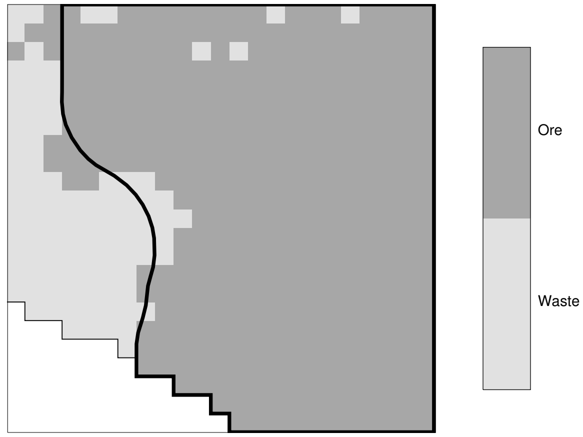

Figure 1.5: Ore/waste classification map. The black line shows the ore/waste boundary for mining.

A mining operation can not economically recover all of the in-situ resources. Some portion of the deposit will be inaccessible or unprofitable to mine. Recoverable reserves are a subset of the resources that can be economically mined.

Assume that we only want to select material with a grade above a specified minimum or cutoff grade, z_c. The cutoff grade can be defined based on many considerations; however, a simplistic formula could be used:

$$z_c = \frac{c_t + (c_o - c_w)}{p \cdot rec} \qquad (1.4)$$

where c_t is the cost of processing the ore ($/tonne), c_o is the cost of mining ore ($/tonne), c_w is the cost of mining waste ($/tonne), p is the metal selling price, and rec is the fraction of metal recovered during processing. The units for metal price are dependent on the units the grade is in. If the grade is oz/tonne, then the metal price needs to be $/oz. The deposit is classified into ore and waste using the cutoff grade.

Histogram Based. Recoverable reserves can be estimated directly from a histogram. The tonnage, quantity of metal, and average ore grade can be calculated. There are two considerations that need to be made when calculating reserves from a histogram: (1) the size of the SMU's that are being used for the reserve calculation, and (2) the volume that the histogram represents. Recall that a distribution changes, the variance and shape, as the support volume changes (Figure 1.3). The drillhole samples have a very small support volume. The blocks that will be mined have a much larger support volume. The distribution needs to be corrected from a "point" scale to a "block" scale. The methodology for the change of support is presented in the next chapter. However, it is important to note that the different sized SMU's will have different block scale distributions, resulting in different reserve estimates. The tonnes of ore can change dramatically with a change in the volume being considered. If the gross tonnage includes areas that will never be mined, the tonnes of ore will be overestimated.

Let T_0 be the total tonnage of the deposit. The recovered tonnage is then:

$$T_A(z_c) = T_0 \cdot [1 - F_{Z_v}(z_c)] = T_0 \cdot \int_{z_c}^{+\infty} f_{z_v}(z)dz \tag{1.5}$$

where $T_A(z_c)$ is the tonnage above the cutoff grade, $f_z(z)$ is the probability density function or frequency distribution of the grades, and $F_Z(z_c)$ is the cumulative distribution function of the grades. The quantity of metal can then be calculated as:

$$Q_A(z_c) = T_0 \cdot \int_{z_c}^{+\infty} z f_z(z)dz \tag{1.6}$$

and the mean grade of the recovered material is given by:

$$M_A(z_c) = \frac{Q_A(z_c)}{T_A(z_c)} \tag{1.7}$$

Only global reserves can be calculated using the histogram based approach. It provides an estimate of the reserves for a given volume where the data is representative.

Block Based. Only SMU's with a grade above the cutoff grade will be mined as ore. This leads to the definition of an indicator function that classifies the SMU as ore or waste according to:

$$i_v(\mathbf{u}_\beta; z_c) = \begin{cases} 1 & \textit{if} \;\; z_v(\mathbf{u}_\beta) > z_c \\ 0 & \textit{otherwise} \end{cases} \tag{1.8}$$

where $\mathbf{u}_\beta$ refers to the SMU centered at location $\mathbf{u}_\beta$, $z_v(\mathbf{u}_\beta)$ is the grade of the SMU, and z_c is the cutoff grade. The tonnes of ore above the cutoff grade for any panel can be calculated using the indicator function and the SMU tonnage:

$$T_V(\mathbf{u}_\alpha; z_c) = \sum_{\beta=1}^{N_v} i_v(\mathbf{u}_\beta; z_c)\, t_v(\mathbf{u}_\beta) \quad \mathbf{u}_\beta \in \mathbf{u}_\alpha \tag{1.9}$$

where $T_V(\mathbf{u}_\alpha; z_c)$ is the tonnage of ore above the cutoff grade for panel V centered at location $\mathbf{u}_\alpha$, $t_v(\mathbf{u}_\beta)$ is the tonnage of the SMU at $\mathbf{u}_\beta$, and $\beta = 1, \ldots, N_v$ where N_v is the number of SMU's contained within the panel, V. The quantity of metal above the cutoff grade is:

$$Q_V(\mathbf{u}_\alpha; z_c) = \sum_{\beta=1}^{N_v} i_v(\mathbf{u}_\beta; z_c) \cdot t_v(\mathbf{u}_\beta) \cdot z_v(\mathbf{u}_\beta) \quad \mathbf{u}_\beta \in \mathbf{u}_\alpha \tag{1.10}$$

The mean grade of the ore is given by:

$$M_V(\mathbf{u}_\alpha; z_c) = \frac{Q_V(\mathbf{u}_\alpha; z_c)}{T_V(\mathbf{u}_\alpha; z_c)} \tag{1.11}$$

The resource estimate for each panel can be upscaled for a global resource estimate.

$$T_A(z_c) = \sum_{\alpha=1}^{N_V} T_V(\mathbf{u}_\alpha; z_c) \quad \mathbf{u}_\alpha \in A$$

$$Q_A\left(z_c\right) = \sum_{\alpha=1}^{N_V} Q_V(\mathbf{u}_\alpha; z_c) \quad \mathbf{u}_\alpha \in A$$

$$M_A\left(z_c\right) = \frac{Q_A(z_c)}{T_A(z_c)}$$

where $\alpha = 1, \ldots, N_V$ and N_V is the number of panels within the deposit, A.

Recoverable reserves calculated from a block model should be consistent with reserves calculated from the appropriate histogram. Block based reserves will take more time to calculate, but more can be done with the block model; e.g. mine planning and selective reserves within a constrained area.

Now that we have seen the factors influencing recoverable reserve calculation, we will discuss the different methods used to estimate reserves.

Chapter 2

Reserve Estimation Methods

The type of reserve estimation required changes at different stages of a mining project. In the early years, global reserves are required for disclosure and financial planning. During the later years, while production is taking place, local reserves are required for mine planning and scheduling. Different methods exist for estimating different reserve types.

2.1 Geostatistical Background

All of the background details for geostatistics cannot be presented in the limited space available here. As much detail as possible will be presented, but the reader is referred to other sources for additional information.

See Isaaks and Srivastava for an excellent introduction to geostatistics including variograms, volume variance, and kriging [8]. Rivoirard gives a complete presentation of non-linear geostatistical methods, including hermite polynomials and uniform conditioning [15]. Goovaerts gives a thorough review of simulation in his book [5]. The book by Chilès and Delfiner is one of the most complete and modern geostatistical references [2]. A recent book presenting the application of simulation to mining problems was written by Journel and Kyriakidis [10]. See Deutsch and Journel for an in depth explanation of a publicly available geostatistical software library [4].

2.2 Global Reserve Estimation

Global reserves are calculated as a preliminary estimate of the reserves and for the calibration of tuned local estimates. At this stage, the location of the reserves within the deposit is not critical. Ensuring that the total recoverable reserves are accurate is the most important criteria at this stage.

It is common to estimate recoverable resources directly from the grade histogram [2]. However, the volume support of the sample data is not the same as the volume support of the blocks that will be mined. A correction needs to be applied to the histogram to account for the larger blocks that will actually be mined. A number of case studies have been published showing that very good results can be achieved with this method [16].

Recall that the variance of blocks within the deposit can be estimated using dispersion variance theory. Change of support models use this block variance to alter the sample histogram so it represents the blocks that will actually be mined. There are three common change of support models: (i) affine, (ii) indirect lognormal, and (iii) discrete Gaussian model (DGM). These models are applied in a similar manner ; however, each change of support model makes different assumptions regarding the shape change of the histogram.

All change of support models make two common assumptions: (1) the variance decreases in a predictable manner as the volume increases, and (2) the mean does not change from one scale to the next. These are both the result of assuming linear averaging, which is correct for fractions or concentrations. Each model assumes a different shape change as the volume increases. These will be developed later in Sections 2.2.2 to 2.2.4. First, the volume variance relationships will be reviewed.

2.2.1 Volume Variance Relations

As the block size increases, the variance decreases. We can define a change of support parameter, or variance reduction factor f, as the variance of the larger volume divided by the variance of the smaller volume:

$$f = \frac{D^2(v, A)}{D^2(\cdot, A)} \tag{2.1}$$

Substituting Equation (1.3) into Equation (2.1) gives:

$$\begin{aligned} f &= \frac{D^2(\cdot, A) - D^2(\cdot, v)}{D^2(\cdot, A)} \\ &= 1 - \frac{D^2(\cdot, v)}{D^2(\cdot, A)} \end{aligned} \tag{2.2}$$

Recall that $D^2(\cdot, A)$ is the variance of the points in the deposit A, or σ^2, and that $D^2(\cdot, v)$ is the variance of the points within the blocks v, or $\overline{\gamma}(v, v)$:

$$f = 1 - \frac{\overline{\gamma}(v, v)}{\sigma^2} \tag{2.3}$$

Therefore, f is a function of the modeled variogram and the variance of the points within the deposit. The variance reduction factor is used directly for the affine variance correction and the indirect lognormal correction. It is used indirectly in the DGM. The spatial continuity of the variable affects the calculated reserves. Reserves for variables that are very continuous will not change substantially as the selection volume increases; whereas, reserves for variables that are not spatially continuous will change drastically as the support volume increases.

2.2.2 Affine Change of Support

The affine change of support model assumes that the shape of the histogram does not change and the variance is reduced by f. The reduction in variance is achieved by:

$$q' = \sqrt{f} \cdot (q - m) + m \tag{2.4}$$

where q is the quantile of the original value in the point scale distribution, m is the mean of the distribution, and q' is the quantile in the corrected block scale distribution.

The assumption that the shape of the histogram does not change is unrealistic for large changes in variance. The affine correction introduces artificial minima and maxima and the shape of the resulting histogram maintains the same asymmetry as the point scale distribution. The affine change of support model is only used for small variance reductions, that is, $f \geq 0.7$ [9].

Recoverable reserves can be calculated using the affine corrected distribution with the histogram based approach from Section 1.1.4.

2.2.3 Indirect Lognormal Correction

The indirect lognormal correction is the change of support model if the point and block distributions were lognormally distributed. The indirect lognormal correction is applied in two steps. The first step is the variance reduction:

$$q' = aq^b \tag{2.5}$$

where q is the original quantile, q' is the variance corrected quantile, and

$$b = \sqrt{\frac{\ln(f \cdot CV^2 + 1)}{\ln(CV^2 + 1)}} \tag{2.6}$$

$$a = \frac{m}{\sqrt{f \cdot CV^2 + 1)}} \left[\frac{\sqrt{CV^2 + 1}}{m} \right]^b \tag{2.7}$$

Where f is the variance reduction factor and CV is the point scale coefficient of variation (σ/m). The second step is to correct the mean of the block distribution back to the mean of the point distribution:

$$q'' = q' \frac{m}{m'} \tag{2.8}$$

where q'' is the final corrected quantile, m is the mean of the original distribution (also the target mean) and m' is the mean after the first correction [8].

The indirect lognormal correction is considered to be more realistic than the affine correction. It assumes an arbitrary shape change and does not impose artificial minima and maxima. Larger reductions in variance can be achieved with the indirect lognormal correction, $f \geq 0.5$.

2.2.4 Discrete Gaussian Model

The discrete Gaussian model (DGM) is also used for change of support at different scales [15]. It is a function defined by a polynomial expansion that needs to be fit to the data. Once the polynomials have been fitted, the function provides a mapping of the point variable Z to the Gaussian variable Y and vice-versa:

$$\begin{aligned} z(\mathbf{u}) &= \Phi(y(\mathbf{u})) \\ &\approx \sum_{n=0}^{np} \phi_n H_n\left[y(\mathbf{u})\right] \end{aligned} \tag{2.9}$$

where np is the highest order term in the polynomial expansion, ϕ_n is a fitted coefficient for each term, and $H_n\left[y(\mathbf{u})\right]$ is the hermite polynomial value defined by the term of the expansion and the y value. Equation (2.9) is referred to as the Gaussian anamorphosis.

Hermite polynomials are defined by Rodrigues' Formula. The polynomial value can be calculated for any value of y and for any polynomial order n using the following formula:

$$H_p(y) = \frac{1}{\sqrt{p!} \cdot g(y)} \cdot \frac{d^p g(y)}{dy^p} \tag{2.10}$$

Where p denotes the order of the polynomial and $g(y)$ is the Gaussian pdf. For example, the first 6 Hermite polynomials are:

$$\begin{array}{ll} H_0(y) = 1 & H_3(y) = -\frac{1}{\sqrt{6}}(y^3 - 3y) \\ H_1(y) = -y & H_4(y) = \frac{1}{2\sqrt{6}}(y^4 - 6y^2 + 3) \\ H_2(y) = \frac{1}{\sqrt{2}}(y^2 - 1) & H_5(y) = -\frac{1}{2\sqrt{30}}(y^5 - 10y^3 + 15y) \end{array}$$

After the zero and first order polynomials have been calculated, the following recursive formula can be used to calculate the higher order polynomials, $H_p(y)$, when $p \geq 2$:

$$H_{p+1}(y) = -\frac{1}{\sqrt{p+1}} y H_p(y) - \sqrt{\frac{p}{p+1}} H_{p-1}(y)$$

We now need to calculate the ϕ coefficients to finish fitting the anamorphosis function. The first order coefficient is:

$$\begin{aligned} \phi_0 &= E\{\phi(y(\mathbf{u}))\} \\ &= E\{z(\mathbf{u})\} \end{aligned} \tag{2.11}$$

or the expected value of $Z(\mathbf{u})$. Higher order ϕ coefficients can be calculated using:

$$\begin{aligned} \phi_p &= E\{z(\mathbf{u}) \cdot H_p(y(\mathbf{u}))\} \\ &= \int \phi(y(\mathbf{u})) \cdot H_p(y(\mathbf{u})) \cdot g(y(\mathbf{u})) \cdot dy(\mathbf{u}) \end{aligned} \tag{2.12}$$

The last expression can be approximated with the data at hand, as a finite summation:

$$\phi_p \approx \sum_{\alpha=2}^{N} (z(\mathbf{u}_{\alpha-1}) - z(\mathbf{u}_\alpha)) \cdot \frac{1}{\sqrt{p}} H_{p-1}(y(\mathbf{u}_\alpha)) \cdot g(y(\mathbf{u}_\alpha)) \tag{2.13}$$

The fitted ϕ coefficients must satisfy the following equality:

$$Var\{Z_{\mathbf{u}}\} = \sum_{n=1}^{np} \phi_n^2 \tag{2.14}$$

where $Var\{Z_{\mathbf{u}}\}$ is the variance of Z at the point support. If the summation is significantly different, the anamorphosis modeling should be checked.

There are two checks that can be done to determine the validity of the modeled anamorphosis function: (1) by comparing the Z to Y transformation function and (2) by comparing the global distribution from the data to the distribution from the anamorphosis. An example of the two plots are shown in Figures 2.1 and 2.2. The

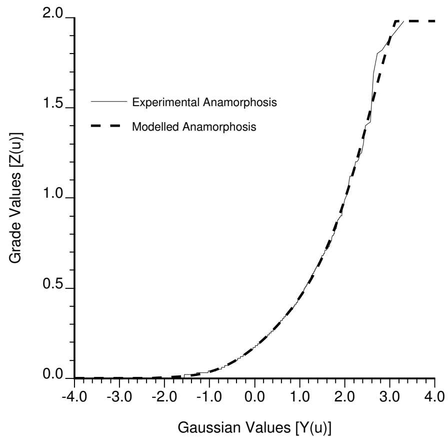

Figure 2.1: Experimental and modeled Gaussian anamorphosis.

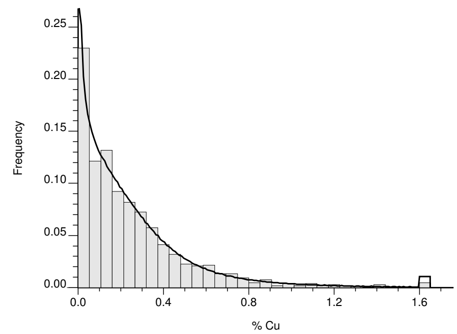

Figure 2.2: Global distribution fit with the Gaussian anamorphosis function..

experimental anamorphosis is shown as a thin line with the modeled anamorphosis shown as a thick dashed line. The original grade distribution is shown as the bar histogram and the fitted distribution is shown as a line. The experimental and modeled anamorphosis functions should be identical, and the properties of the two distributions should be similar.

The discrete Gaussian model is used for estimating the change of support from point scale to block scale. The variability and shape of the block scale distribution is controlled by the anamorphosis.

This anamorphosis function (Equation 2.9) can be modified to account for the change of support from point data to block data by the addition of a change of support coefficient r:

$$\begin{aligned} z(v) &= \Phi(y(v)) \\ &\approx \sum_{n=0}^{np} r^n \phi_n H_n\left[y(v)\right] \end{aligned} \tag{2.15}$$

By calculating the value of r, we can determine the distribution of grades for volumes of support larger than the point samples.

The calculation of r requires the variance of the larger support volumes. Typically, there is not enough data available to do this explicitly. The variance of the larger blocks can be estimated using the modeled variogram of the point data; recall Equation (1.3).

$$D^2\left(v, A\right) = D^2\left(\cdot, A\right) - D^2\left(\cdot, v\right)$$

where v is the SMU support volume and $\cdot$ is the point support volume. Therefore we can calculate the variance of the larger blocks using the modeled variogram. Recall Equation 2.14:

$$Var\left\{Z.\right\} = \sum_{n=1}^{np} \phi_n^2$$

Since this equality must be true for the point support, it must also be true for the block support. By including r in the equality, the equation becomes:

$$Var\left\{Z_v\right\} = D^2\left(v, A\right)$$

$$
\begin{aligned}
&= D^2(\cdot, A) - D^2(\cdot, v) \\
&= \sum_{n=1}^{np} r^{2n}\phi_n^2
\end{aligned}
\tag{2.16}
$$

where $Var\{Z_v\}$ is the variance of Z at the SMU support. The only unknown parameter is r. Any optimization method can be employed to find the value of r.

After calculating the change of support coefficient, the block scale distribution can be estimated. Global reserves are then estimated using the upscaled block distribution. The discrete Gaussian model is a fairly robust upscaling algorithm. It assumes a permanence of Gaussianity in Y space. As a result, the distribution tends to a Gaussian shape as the variance at the block scale decreases.

Global reserve estimation has proven to be fairly robust. The estimated grade and tonnage above a cutoff is usually close to the results seen during mining. Local estimates are usually tuned using a global reserve estimate. The local estimation method is tuned so that the reserves match the reserves from the global method. Tuning the local estimates is discussed after the local reserve estimation methods have been discussed.

2.3 Local Reserve Estimation

Although global resource estimation techniques can be used at initial mine evaluation stages, they cannot be used when considering local areas in the mine. Pit optimization and short/long term planning require local estimation of the grades. Global reserves can be calculated using the local estimates and compared with the global reserves calculated previously.

Local resources or reserves are calculated using an estimation technique, such as ordinary kriging, or a simulation technique, such as sequential Gaussian simulation. The different methods and implementation suggestions are presented below.

2.3.1 Kriging

Kriging is a well-established estimation method. The estimates are calculated as a weighted combination of the data. The weights are chosen to minimize error variance. There are some limitations to kriging. The first major consideration is that the estimates are smoother than the underlying true grades and this smoothness depends on the data spacing. Secondly, kriging does not provide a good measure of uncertainty. The kriging variance depends only on the data configuration and the variogram values. The magnitude of the data values and the shape of the distribution are not considered when calculating the kriging variance.

Kriging can be used to estimate the grade of the SMU sized blocks. These block estimates are used for reserve calculation. It is a fast way to get recoverable reserves. However, the smoothing makes the calculated reserves unreliable [9].

A kriged estimate is a linear combination of the surrounding data values:

$$
z^*(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i \cdot z(\mathbf{u}_i) + \left[1 - \sum_{i=1}^{n} \lambda_i\right] \cdot m \tag{2.17}
$$

where $\mathbf{u}$ refers to a location in space, $z^*(\mathbf{u})$ is the estimate at location $\mathbf{u}$, $z(\mathbf{u}_i)$ is data at location $\mathbf{u}_i$ are there are n data values, λ_i is the weight assigned to the i^{th} data value, and m is the global mean. This equation is simplified by working with residuals:

$$y(\mathbf{u}) = z(\mathbf{u}) - m$$
$$y^*(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i \cdot y(\mathbf{u}_i) \tag{2.18}$$

We need to solve for the kriging weights before the estimate can be calculated. The weights are calculated by minimizing the kriging error variance. The kriging variance is:

$$\begin{aligned}
\sigma_E^2 &= E\left\{\left[Y^*(\mathbf{u}) - Y(\mathbf{u})\right]^2\right\} \\
&= E\left\{\left[Y^*(\mathbf{u})\right]^2\right\} - 2 \cdot E\left\{Y^*(\mathbf{u}) \cdot Y(\mathbf{u})\right\} + E\left\{\left[Y(\mathbf{u})\right]^2\right\} \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j E\left\{Y(\mathbf{u}_i) \cdot Y(\mathbf{u}_j)\right\} - 2 \cdot \sum_{i=1}^{n} \lambda_i E\left\{Y(\mathbf{u}_i) \cdot Y(\mathbf{u})\right\} + C(0) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j C_{ij} - 2 \cdot \sum_{i=1}^{n} \lambda_i C_{i0} + \sigma^2
\end{aligned} \tag{2.19}$$

where 0 refers to the unsampled location, C_{ij} is the covariance between data points i and j, C_{i0} is the covariance between the i^{th} data point and location being estimated, n is the number of data points, and σ^2 is the variance of the data. The optimal weights are determined by taking the partial derivative and setting them to zero:

$$\frac{\partial \left[\sigma_E^2\right]}{\partial \lambda_i} = 2 \cdot \sum_{j=1}^{n} \lambda_j C_{ij} - 2 \cdot C_{i0} \tag{2.20}$$

$$\sum_{j=1}^{n} \lambda_j C_{ij} = C_{i0}, \quad i = 1, \ldots, n \tag{2.21}$$

Solving for the weights allows the estimate at $\mathbf{u}$ to be calculated. The estimation variance is a measure of uncertainty in the estimate at $\mathbf{u}$.

Many flavors of kriging have been developed for different problems: (1) simple kriging, (2) ordinary kriging, (3) multi-Gaussian kriging, (4) lognormal kriging, (5) indicator kriging, and more. All of the available kriging methods use the same underlying principals for calculating an estimate; minimizing the error variance. The difference between the kriging types, is the transformation that is applied to the data before and after the estimation and the constraints imposed during the minimization. The readers are referred to the references listed earlier for additional information.

2.3.2 Uniform Conditioning

Kriging small blocks in relation to the data spacing can be problematic. However, we can estimate larger blocks, called panels, with kriging and get reliable estimates. Uniform conditioning is a technique that predicts the distribution of the SMU grades within each estimated panel [15]. In other words, given that we know the grade of

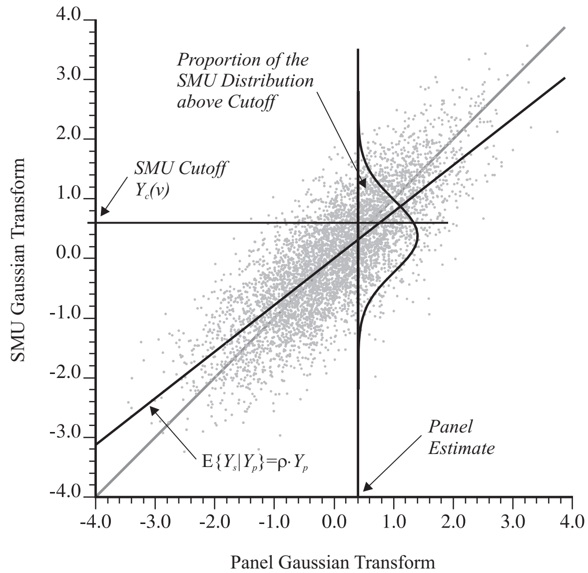

Figure 2.3: Conditional distribution of the smu grades within a panel given the panel estimate [17].

a large block, the panel that has been estimated, we can calculate the distribution of the SMUs that are in the panel.

Uniform conditioning uses the discrete Gaussian model to perform the change of support. Refer to Section 2.2.4 for the details of the discrete Gaussian model. Recall the change of support parameter r that was used to upscale the sample distribution to the SMU sized blocks. Another change of support parameter, r', is required for the panels. The two change of support parameters can be used to upscale the sample distribution to the SMU or panel support volumes. In addition, they can be used to downscale the panel estimate to an SMU scale distribution.

Given that the panel grade is known, the distribution of SMUs within the panel can be calculated. The average of the SMUs is the panel grade, and the variance is based on the change of support coefficients. For a panel grade, $y(V)$, the SMUs within that panel will have a mean and variance of [15]:

$$E\{y(v)\} = \frac{r'}{r} \cdot y(V)$$
$$Var\{y(v)\} = 1 - \left(\frac{r'}{r}\right)^2$$

Consider the panel estimate and SMU distribution in Figure 2.3. The recoverable reserves are defined by the proportion and quantity of metal above the cutoff grade. These are easily calculated using the bivariate Gaussian assumption and the SMU anamorphosis function. The conditional expectation line in Figure 2.3 is defined analytically from the bivariate Gaussian distribution between the Gaussian transforms of the SMU and panel grades. The fact that the slope is not 45 degrees is not an indication of conditional bias.

The proportion above the cutoff grade is [14, 15]:

$$\begin{aligned} P(z_c) &= P\left[z(v) \geq z_c | z(V)\right] \\ &= P\left[y(v) \geq y_c | y(V)\right] \end{aligned}$$

$$= \quad 1 - G\left(\frac{y_c - \left(\frac{r'}{r}\right) y(V)}{\sqrt{1 - \left(\frac{r'}{r}\right)^2}}\right) \tag{2.22}$$

The quantity of metal can be calculated in one of two ways. The first is an integration of the conditional distribution above the cutoff grade [14]:

$$Q(z_c) \quad = \quad \int_{y_c}^{\infty} \Phi_v(y(v)) g(Y(v)|Y(V)) d(y(v)) \tag{2.23}$$

The second is by using the fitted hermite polynomials [15]:

$$\begin{aligned} Q(z_c) &= E\left[Z(v) I_{Z(v) \geq z_c} | Z(V)\right] \\ &= E\left[\Phi_v(Y(v)) I_{Y(v) \geq y_c} | Y(V)\right] \\ &= \sum_{n=0}^{\infty} q_n \left(\frac{r'}{r}\right)^n H_n(Y(V)) \\ &= \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \phi_p r^p U_p^n(y_c) \left(\frac{r'}{r}\right)^n H_n(Y(V)) \end{aligned} \tag{2.24}$$

where

$$\begin{aligned} U_0^0(y_c) &= 1 - G(y_c) \\ U_k^0(y_c) &= U_0^k(y_c) = \frac{-1}{\sqrt{k}} H_{k-1}(y_c) g(y_c) \\ U_p^n(y_c) &= \frac{-1}{\sqrt{n}} H_p(y_c) H_{n-1}(y_c) g(y_c) + \sqrt{\frac{p}{n}} U_{p-1}^{n-1} \end{aligned}$$

The grade above cutoff is calculated using the estimated quantity of metal and proportion above cutoff:

$$M(z_c) = \frac{Q(z_c)}{P(z_c)}$$

Note that this equation becomes unstable as the proportion of the block above cutoff decreases below 1%.

There are limitations to the uniform conditioning method. One is that it does not provide any spatial location for the SMUs contained in each panel. The high grade SMUs and the low grade SMUs could be anywhere. The second limitation is that two panels with the same estimated grade will have the same reserves irrespective of surrounding data.

2.3.3 Conditional Simulation

The local reserve estimation techniques presented above provide an estimate of the reserves. They do not provide a distribution of uncertainty. Conditional simulation allows us to generate multiple realizations for assessing global and local uncertainty in our model.

Simulation overcomes two limitations of kriging: (1) it corrects for the smoothing effect, and (2) it allows the joint uncertainty in the model to be assessed. Simulation reproduces the correct spatial variability by adding a random component to an estimated value to get a simulated value and it uses the simulated values as conditioning points as the simulation progresses. By generating multiple realizations, the model uncertainty can be assessed at any scale from a single block to the entire deposit. In addition, the uncertainty for a mine plan can be calculated. For a given volume, such as a pit or production interval, the range of uncertainty is easy to assess.

The smoothing effect of kriging makes the variance of kriged estimates too small. The variance of a kriged estimate is [5]:

$$Var\left\{y^*\left(\mathbf{u}\right)\right\} = \sigma^2 - \sigma_E^2$$

where $y^*\left(\mathbf{u}\right)$ is the kriged estimate at $\mathbf{u}$, σ^2 is the variance of the data and σ_E^2 is the kriging variance. The variance of the estimates can be corrected by adding the missing variance to the simulated value:

$$y_S\left(\mathbf{u}\right) = y^*\left(\mathbf{u}\right) + R\left(\mathbf{u}\right)$$

Where $y_S\left(\mathbf{u}\right)$ is the simulated value at $\mathbf{u}$ and $R\left(\mathbf{u}\right)$ is a random residual with a mean of 0.0 and a variance equal to the σ_E^2. The variance of the simulated values is now correct. The covariance between simulated values can be made correct by performing the simulation in a sequential fashion. The cells in the model are simulated in a random order, and the previously simulated cells are used as conditioning data when simulating subsequent cells in the model [5].

Sequential Gaussian simulation is a commonly used simulation algorithm, It is used to simulate continuous variables under a Gaussian paradigm. The steps required to build a simulated model are [4]:

1. transform the data to Gaussian units with a normal score transformation,
2. assign the data to the nearest grid node in the model,
3. determine a random path through all of the grid cells to be simulated,
4. construct the conditional distribution at a location using the nearby samples and previously simulated grid cells,
5. draw a simulated value from the conditional distribution,
6. add the simulated value to the pool of data,
7. go to the next node and repeat steps 4, 5, and 6 until all cells have been simulated,
8. back transform the realization
9. go back to step 3 to generate another realization.

There are several other types of simulation: (1) turning bands, (2) LU decomposition, (3) probability-fields, (4) sequential indicator simulation, (5) direct sequential simulation, and more. The readers are referred to the references listed earlier for additional information.

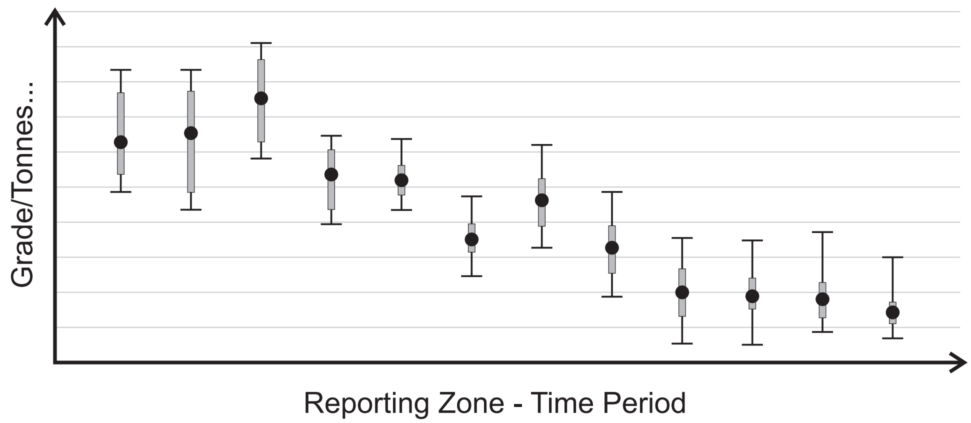

Figure 2.4: Uncertainty in mine production for different time periods [3].

2.3.4 Upscaling Local Reserves

Aside from local distributions of uncertainty, the move towards simulation from traditional estimation methods allows one other advantage. A realization of simulated values can be directly upscaled to any arbitrary volume and the change in resources/ reserves evaluated directly. No explicit change of support model is required, hence no distributional assumptions are imposed.

Upscaling in this instance generally involves taking the simulated grid and averaging all relevant blocks within the volume of interest. For instance, if the panel volume represents a monthly production period, only those blocks within the production area are averaged. As before, the panel volume need not be regular, polygons may be considered, that may not be contiguous regions, representing multiple advancing faces in production. This averaging of multiple simulated grades yields the panel grade. Repeating this averaging process over multiple realizations yields a distribution of uncertainty in the panel grade; thus, the uncertainty of a panel grade can be directly obtained.

Local resources and reserves can be upscaled to any arbitrary volume, not just a larger panel. Monthly, quarterly, annual, or global reserves can be calculated from a small scale model. Upscaling multiple realizations allows the uncertainty in the larger production volumes to be calculated as well; see Figure 2.4. In practice, the biggest limitation to this approach is the computational effort to generate and store these multiple realizations.

2.4 Tuned Estimation

HERCO analysis, Hermite Polynomial correction, is used to tune the kriging parameters so that the distribution of the estimated blocks matches the upscaled global distribution from the hermite polynomial change of support [6]. The first step is to choose an SMU block size and upscale the distribution using the Discrete Gaussian model. The second step is to perform a kriging of the SMU blocks with the data at hand. The results of the DGM and the kriging are then compared. If there are any discrepancies, the kriging parameters are modified until the kriging results match the results from the DGM. The usual modification made to the kriging parameters is to restrict the search and the number of samples used. This has become common practice and is referred to as Restrictive Kriging.

The advantage of a restrictive search is that the global reserves are correct. By restricting the search, the smoothing effect of kriging is mitigated and the distribution of the estimates can be made to resemble the upscaled distribution. The disadvantage is that restrictive kriging introduces significant conditional bias. Areas that are estimated high will never be that high at the time of mining and areas that are estimated low will never be that low when they are mined. The aim is to get the global reserves correct.

Another tuning scheme was proposed to correct the smoothing effect of kriging [1]. The correction adjusts the variance of the kriging estimates to match the variance of the SMUs. For example; suppose that the SMUs have a variance of:

$$D^2(v, A)$$

The smoothing effect of kriging results in the SMU estimates having a variance less than the true variance:

$$\sigma^2_{z_v^*} < D^2(v, A)$$

The variance of the estimates needs to be increased by the SMU estimation variance:

$$\begin{aligned} \sigma^2_{z_v^*,cor} &= \sigma^2_{z_v^*} + \sigma^2_{e,z_v^*} \\ &\approx D^2(v, A) \end{aligned} \tag{2.25}$$

where $\sigma^2_{z_v^*,cor}$ is the variance of the corrected SMU estimates, $\sigma^2_{z_v^*}$ is the variance of the kriged SMU estimates, and σ^2_{e,z_v^*} is the kriging error variance for the SMU estimates. Correcting the estimates to a target dispersion variance is very similar to performing the HERCO analysis and restrictive kriging. It aims to get the global reserves correct.

In open pit mining, it is important to get the reserves correct at the early stages of mine development. During mining, all of the material contained within the pit will be mined and additional grade control samples will be available to classify the ore and waste. The conditional bias will not be significant and the recovered reserves will closely match the reserves estimated with the conditionally biased model. The same cannot be said of underground mines. Stopes are planned and mined essentially unchanged by additional information available in the future. It is important to get the local reserves correct for underground mines; in other words, the estimates must be conditionally unbiased [1].

Chapter 3

Comparative Case Study of Reserve Estimation Techniques

A full case study was prepared to demonstrate the complex, site-specific considerations. The data set was provided by Placer Dome Technical Services. The particulars of the mine will be kept confidential; however, the material in this Chapter is published with the permission of Placer Dome.

Four methods were used to calculate recoverable reserves for the case study: (1) change of support models, (2) ordinary kriging, (3) uniform conditioning, and (4) conditional simulation. A grade tonnage curve will be calculated from the results of each method. The different grade tonnage curves will be compared.

The SMU size for the study is 10x10x10m, and the panel size, for uniform conditioning, is 60x60x20m. The following steps were done as part of the reserve estimation:

- exploratory data analysis,
- spatial continuity analysis,
- direct estimation of the global resources,
- kriging for global resources,
- uniform conditioning for global resources,
- conditional simulation for global resources, and
- comparison of the results.

The results from each step are documented below and issues discussed.

3.1 Data

The topography model of the mine was used to get a better understanding of the mining operation. The topography is shown in Figure 3.1. The composited drillhole data contained the x, y, and z locations of each composite along with the gold and silver grades. The drillhole traces are shown in Figure 3.2. Figure 3.3 shows the drillhole data with the mine topography. Figure 3.4 shows the samples for bench

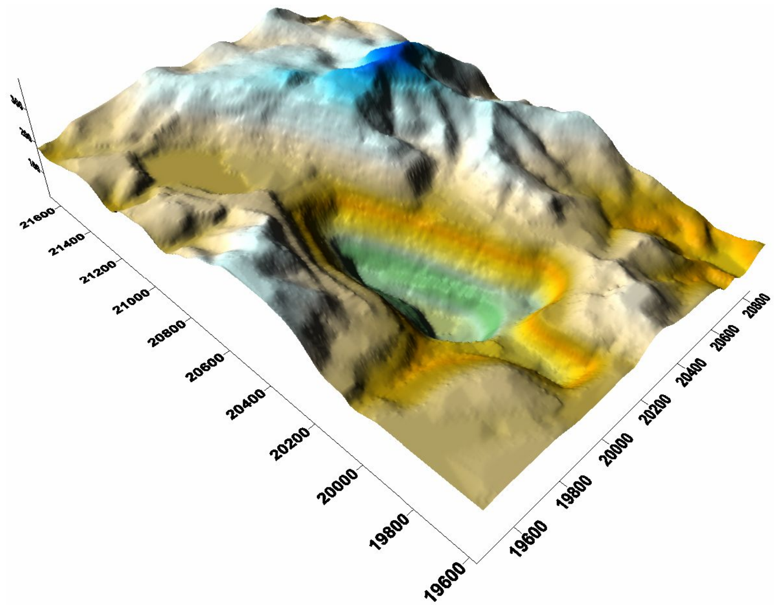

Figure 3.1: Topography of the existing mine.

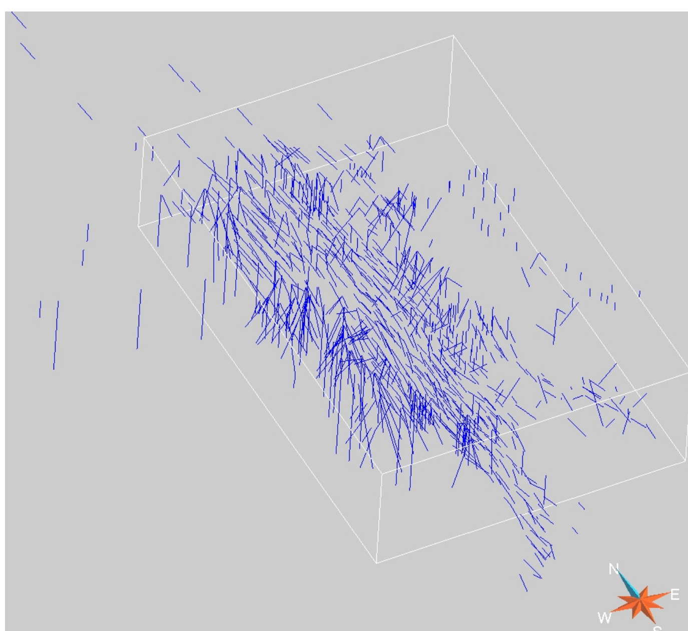

Figure 3.2: Drillhole data set.

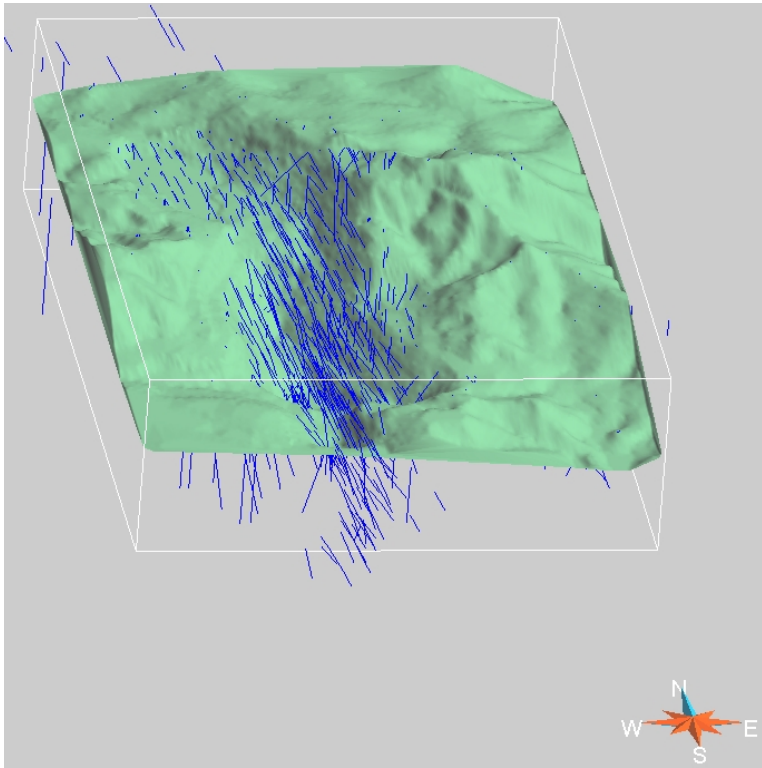

Figure 3.3: Topography of the mine with the drillholes.

200. It is easy to see on the graph that the higher grade areas are located towards the middle of the deposit with the lower grades at the extremities. Figure 3.5 shows a vertical cross section of the deposit.

3.2 Exploratory Data Analysis

The goal of the exploratory data analysis is to get representative statistics for the variables in the deposit. The important mineral for this study is gold.

The average gold grade is 0.44 grams/tonne (g/t) with a variance of 1.66. Figure 3.6 shows the gold histogram. The histogram on the left has an arithmetic axis, while the histogram on the right has a lognormal axis.

Samples are almost always collected in a spatially biased fashion. Declustering is used to determine statistics that are representative for the entire deposit from the biased samples. The sample spacing is relatively close near the center of the deposit. It is approximately 50m. Near the outside of the deposit the sample spacing increases to approximately 150m. A declustering cell size of 150m was chosen using the plots in Figure 3.7 and the average data spacing in the sparsely sampled areas. The declustering results are shown in Figure 3.8. As expected, the mean and variance of the gold grade decreased.

Sequential Gaussian simulation and the discrete Gaussian model for change of support require the data to be standard normal. This means the data must follow a normal distribution with a mean of zero and a variance of one. The gold grades

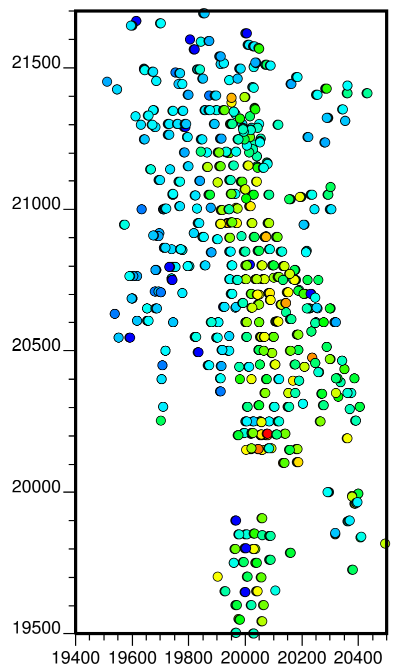

Figure 3.4: Plan view of samples for bench 200 (Low grade samples are blue and high grade samples are red).

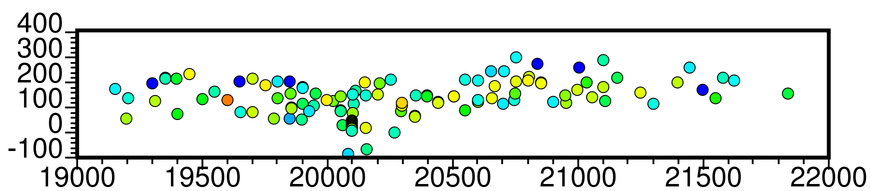

Figure 3.5: Vertical cross-section looking west at 2000m Easting.

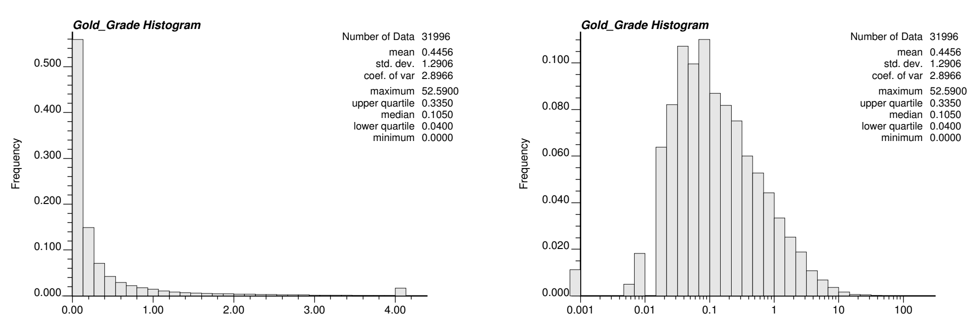

Figure 3.6: Gold grade histograms.

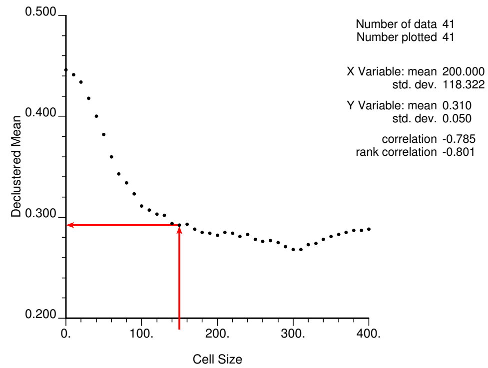

Figure 3.7: Declustered mean versus cell size.

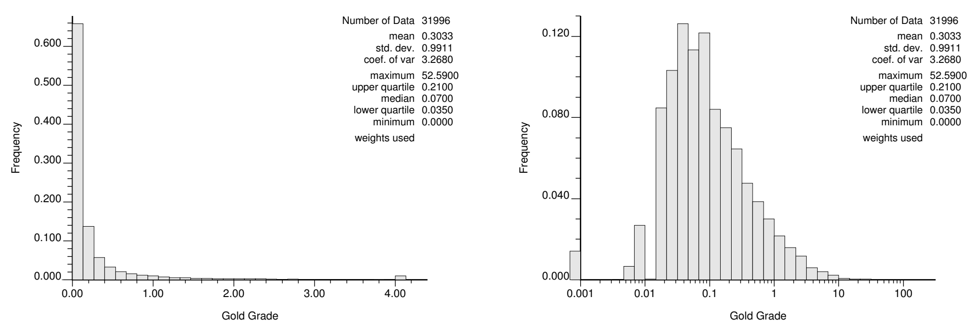

Figure 3.8: Declustered gold grade histogram.

were transformed to normal scores twice. The first time the declustering weights were not used. These normal score values will be used for calculating variograms. The declustering weights were used for the second transformation. These normal score values will be used for the conditional simulation.

3.3 Variogram Analysis

The real space experimental variograms showed pure noise; changes to the lag spacing, tolerance, or minimum number of pairs made little difference. Figure 3.9 shows the poor quality of the real space variogram maps. The spatial continuity is much easier to see in the normal score variogram maps (Figure 3.9). The major direction of continuity was chosen as -10° azimuth from north, the minor direction is 80° azimuth from north and the vertical variogram is straight down.

Since the calculated variograms for the original gold variable were pure nugget, some alternate measures of spatial continuity were evaluated. Specifically, correlograms, pairwise relative and the covariance were analyzed. The correlogram was the most stable measure and it was used for modeling the real space gold variogram (see Figure 3.10):

$$\gamma(\mathbf{h}) = 0.25 + 1.12 \cdot Exp_{\substack{hmax=85\\hmin=60\\hvert=17}}(\mathbf{h}) + 0.3 \cdot Sph_{\substack{hmax=440\\hmin=120\\hvert=250}}(\mathbf{h})$$

The normal score variograms were much better behaved than the real space variograms. There was no need to consider a robust variogram measure. The

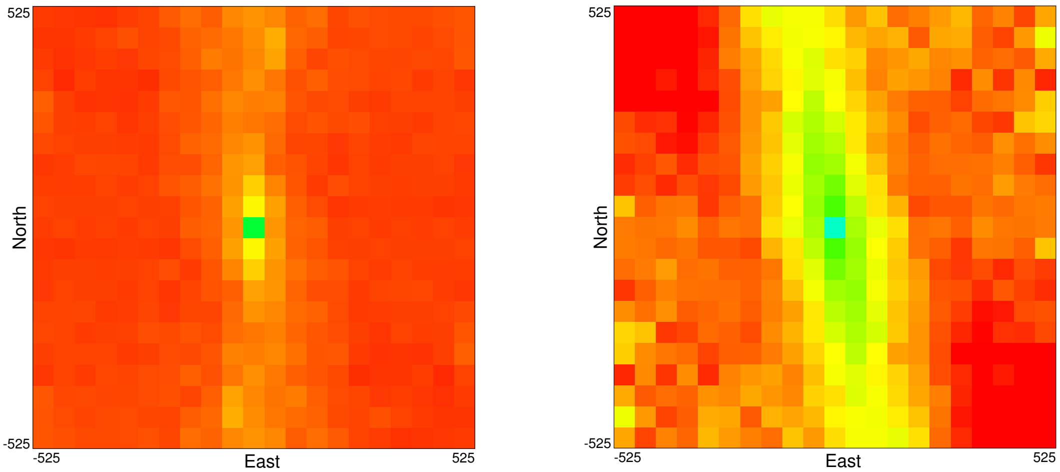

Figure 3.9: Real space and normal score variogram maps. The real space variogram map is shown on the left and the normal score variogram map is shown on the right.

experimental normal score variogram is shown in Figure 3.10. The normal score gold variogram model is:

$$\gamma(\mathbf{h}) = 0.15 + 0.52 \cdot Exp_{\substack{hmax=60\\hmin=55\\hvert=90}}(\mathbf{h}) + 0.3 \cdot Sph_{\substack{hmax=1300\\hmin=350\\hvert=270}}(\mathbf{h})$$

3.4 Direct Estimation of the Global Resources

The variance of blocks within the deposit was estimated using the modeled real space variogram. The gammabar value of the SMU sized blocks is 0.452, the estimated variance of the blocks within the deposit is 0.530, and the variance reduction factor for the change of support calculation is 0.540.

These values were used to estimate the block scale distribution through the three change of support models previously documented. The results of the affine change of support are shown in Figure 3.11, the results for the indirect lognormal change of support are shown in Figure 3.12, and the results for the discrete Gaussian model are shown in Figure 3.13. All of the figures have the original gold grade histogram on the left and the variance corrected histogram on the right. The affine corrected histogram shows the artificial minimum and maximum that are introduced as part of the correction. The distribution tends towards the mean to reduce its variability. The indirect lognormal correction and the discrete Gaussian model do not impose an artificial minimum and maximum compared to the affine correction. The histograms from the indirect lognormal correction and the discrete Gaussian model appear more realistic than the affine corrected histogram.

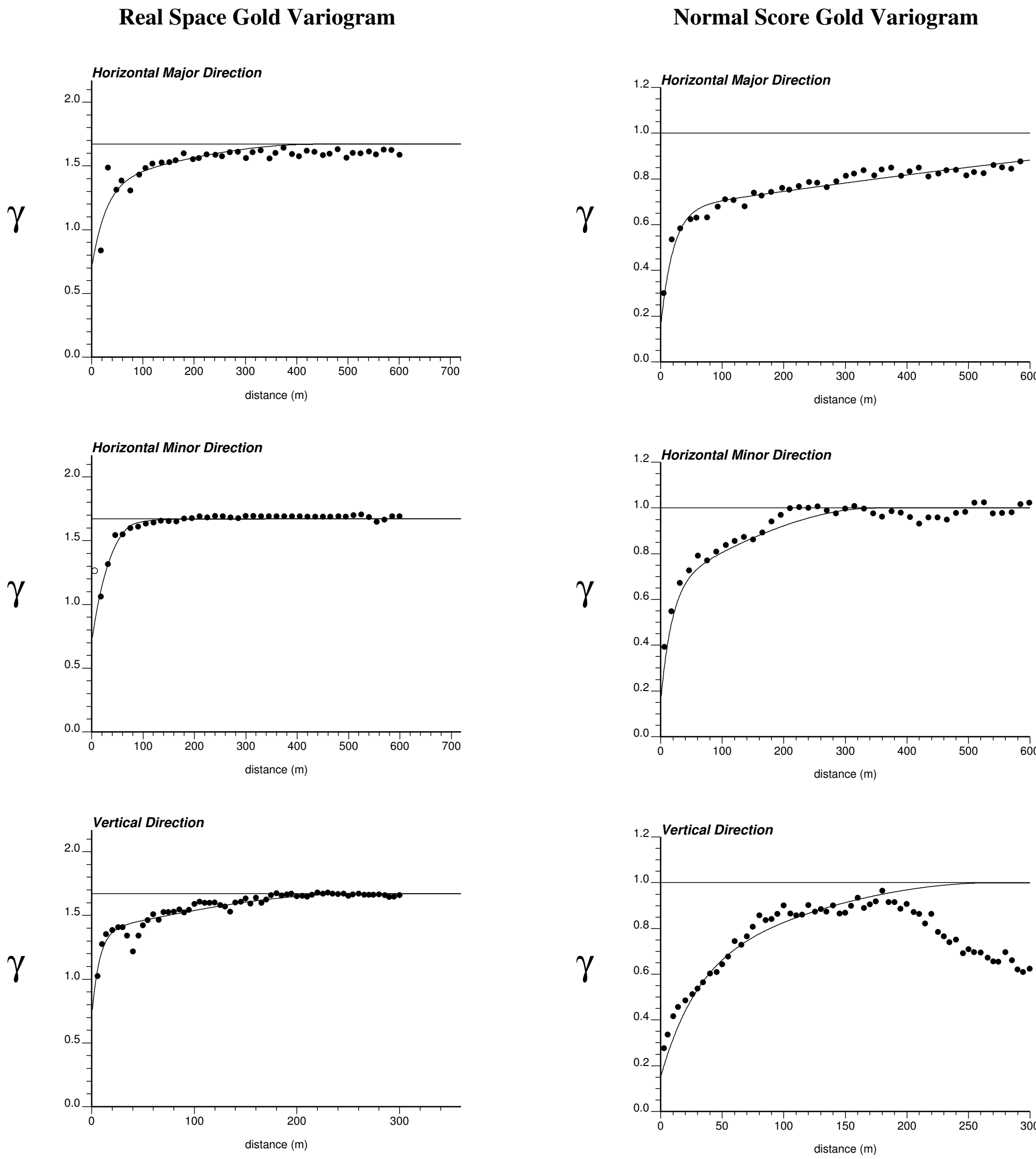

Figure 3.10: Real space and normal score variogram models. The real space variogram model is shown on the left and the normal score variogram model is shown on the right.

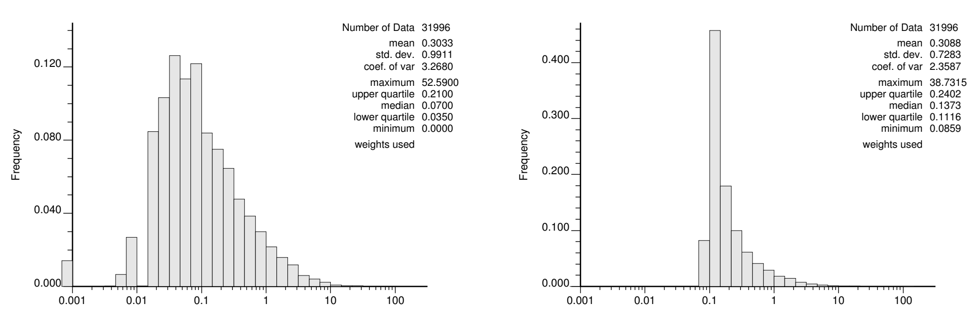

Figure 3.11: Affine corrected histogram.

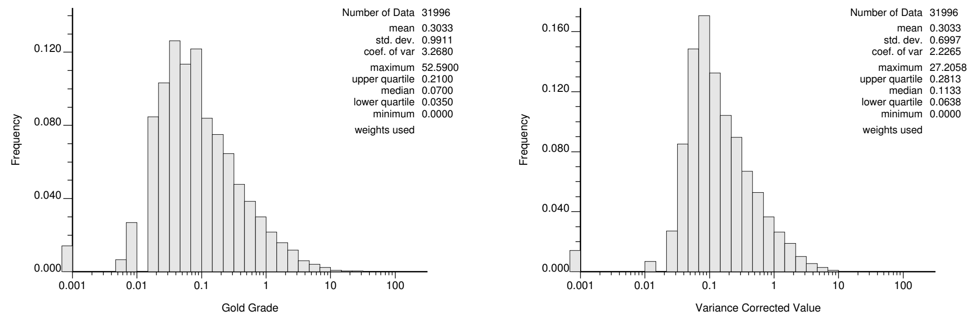

Figure 3.12: Indirect lognormal corrected histograms.

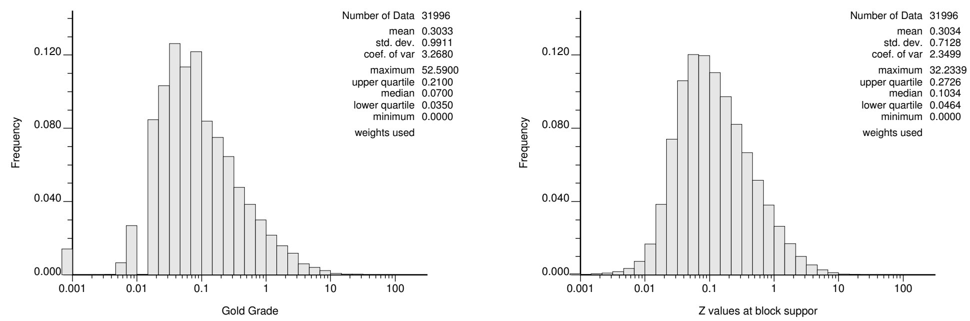

Figure 3.13: Discrete Gaussian model corrected histograms.

3.5 Kriging for Global Resources

Ordinary kriging was used to estimate the grade for the SMUs directly. This was not expected to give good results but was done for comparison purposes. Figure 3.14 shows the kriged SMU gold grades. The kriged panel estimates are shown in Figure 3.15. The panel estimates will be used for the uniform conditioning.

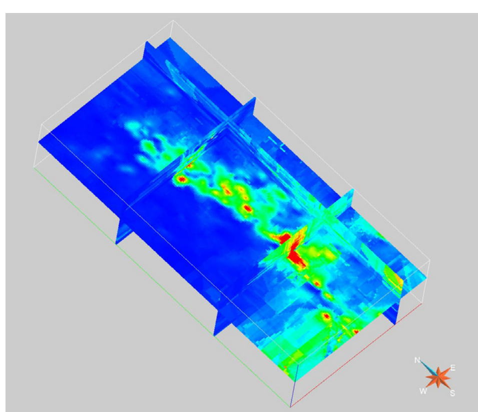

Figure 3.14: Ordinary kriging of the SMU's directly.

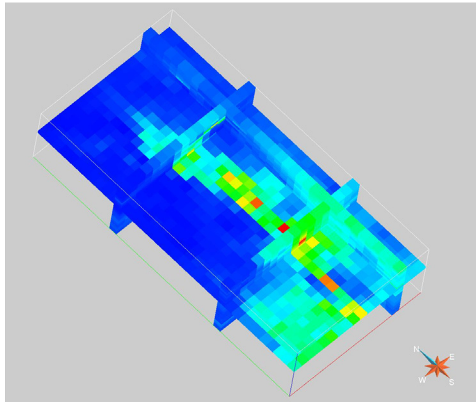

Figure 3.15: Ordinary kriging of the panel grades for uniform conditioning.

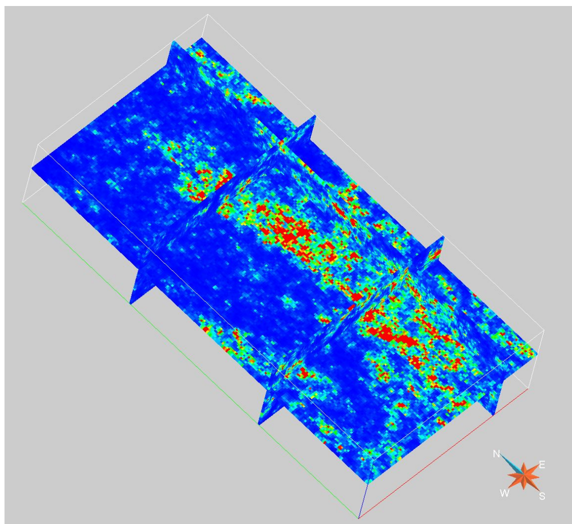

Figure 3.16: Back transformed simulation, block averaged to the SMU blocks.

3.6 Uniform Conditioning for Global Resources

The panel estimates were used to calculate reserves with uniform conditioning. The reserves were calculated for each panel and the results upscaled to provide global reserves.

3.7 Conditional Simulation for Global Resources

Sequential Gaussian simulation was used to build conditional simulations of the gold grade. The first step was to generate 51 realizations of the gold grade at a fine scale, and then upscale the results to the SMU blocks. Simulating to a very small block and then block averaging up to the larger SMU sized blocks allows for the change of support to be accounted for directly. Figure 3.16 shows one realization of the block averaged gold grades.

3.8 Comparison of the Results

Now we can calculate and compare the grade tonnage curves for the different methods. The grade tonnage curves for gold are shown in Figure 3.17. The solid black line shows the average grade tonnage curve for the simulation results, the dashed black line shows the 90% probability interval (P05-P95) from simulation and the other colored lines represent the other methods. A number of important observations can be made: (1) the simulation results match very closely with the uniform conditioning results, (2) the discrete Gaussian and indirect lognormal change of support models are very similar to the simulation results, and (3) the affine change of support and the ordinary kriging results are poor. The change of support models are dependent on the parameters that define the change in variance, and thus can change dramatically with a small change in the variogram. In other words, the change of support models could be made to match the simulated results even better with revised change of support parameters. Figure 3.18 is a close up of the grade tonnage curve.

For interest sake, the e-type from the simulation results (that is the local average over all realizations) were also considered and the corresponding grade tonnage curve calculated. The grade tonnage curve is shown in Figure 3.19. Note that the e-type grade tonnage curve is very similar to the kriging curve. It does not provide a good estimate of the recoverable resources. Rather than averaging the grades and then calculating the grade tonnage curve (as done here), one should calculate the grade tonnage curve associated to all realizations, and then take the average of the grade tonnage curves (as illustrated by the black line on the plots).

The case study presented in this chapter made several assumptions. Most notably was a fixed SMU size. The SMU was arbitrarily chosen for this study. Since the SMU size has a significant impact on the reserve calculation, any resource study should carefully select the SMU size. Methodology for choosing an SMU size is presented in the next 2 Chapters.

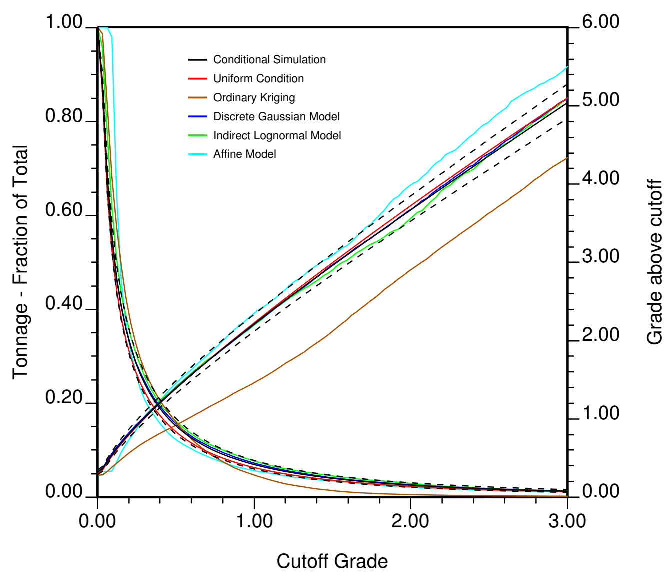

Figure 3.17: Gold grade tonnage curve.

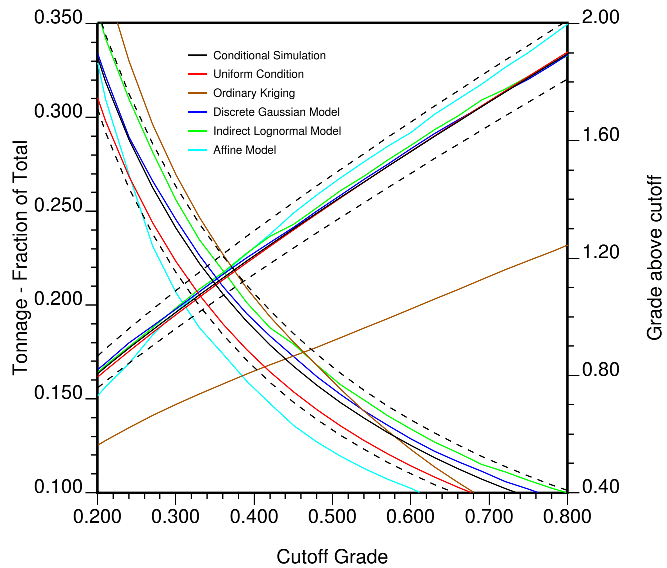

Figure 3.18: Close up of the gold grade tonnage curve.

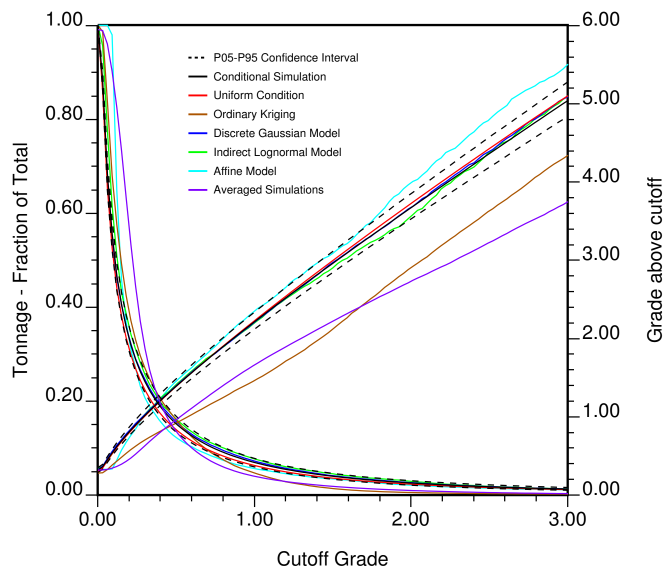

Figure 3.19: Average simulation grade tonnage curve or average grade tonnage curve.

Chapter 4

Optimal Selection of Selective Mining Unit Size

The calculation of mineral resources and ore reserves from a block model requires the choice of a block of selective mining unit (SMU) size. Each block is assigned a grade or a distribution of grades. The resources/reserves are calculated from these block values. Selecting an appropriate SMU size requires consideration of the mining equipment, bench height, blasthole sampling, grade control practice, and affect of dilution. The use of large mining equipment will preclude use of an SMU size that is smaller than the volume of material that can be extracted by the equipment. Dilution is another important consideration that will affect the tonnes and grade of ore. Conventional grade control practice traditionally involves using information from blasthole samples and on-site visual inspections to refine the ore-waste boundary. We do not attempt to address the use of visual controls on grade control. Clearly, if there are visual controls in the pit they must be considered.

This chapter proposes a methodology to determine the optimal SMU size to match actual production. Actual production is simulated on a representative area by simulating the collection of blasthole data and the consequent grade control. Then, the geostatistical resource estimation procedure is implemented for a range of SMU block sizes and the SMU size that gives a reasonable match to the actual production is recommended. The optimal SMU size will yield an estimate of tonnes of ore and grade of ore that is close to the actual production. An example is shown to illustrate the methodology. Different types of dilution and the affect of grade control data sampling on the SMU size are also discussed. [1]

4.1 Introduction

The conventional definition of the selective mining unit (SMU) is the smallest volume of material on which ore waste classification is determined [19]. The reality is more complex. It is impractical and impossible to freely select an SMU of ore in the midst of waste just as it is impossible to freely reject an SMU of waste in the midst of ore. Nevertheless, even large bulk mining equipment may have the ability to mine within a couple of meters of a boundary if the conditions are favorable. The

[1] A version of this chapter has been accepted for publication [12].

SMU size depends on a number of different factors, including the mining equipment size, the mining method to be used, the direction of mining, and the depositional environment of the orebody.

Our unconventional definition of the selective mining unit (SMU) is the block model size that would correctly predict the tonnes of ore and diluted head grade that the mill will receive with anticipated grade control practice. Mutual exclusion of ore and waste material means that correctly predicting the tonnes of ore entails correct prediction of the tonnes of waste. This size must somehow be related to the ability of the equipment to select material, but it is also based on the data available for classification (blastholes and/or dedicated grade control drilling), the procedures used to translate that data to mineable dig limits, and the efficiency with which the mining equipment excavates those dig limits. Numerous sources of dilution must also be accounted for including internal dilution due to grade variability within the SMU, external dilution resulting from geological/geometric contacts, and operational dilution that accounts for production errors, pressures and schedule demands.

Conventional grade control practice uses information from blasthole samples and on-site visual inspections to refine the ore-waste boundary. We do not attempt to address the use of visual controls on grade control. Clearly, if there are visual controls in the pit they must be considered. A common way of translating blasthole data to dig limits is the outline-and-average method where ore or waste regions are delineated by a polygon that implicitly accounts for the equipment. Kriging is sometimes used to improve on the border between ore and waste. Simulation and loss functions have gained limited use in further refining the boundaries between ore and waste. The available data is a clear limitation to the resolution with which we can pick limits. Dedicated grade control sampling or closer spaced smaller diameter blastholes provide some refinement, but a cost-benefit analysis must be performed.

In practice, the tonnes/grade of ore that the mill receives is the result of a classification procedure with many subjective factors. The mill certainly does not receive the values in a long- or medium-term block model. There is much more information at the time of mining and the blocks are never freely and perfectly selected in any case. It would be impractical with existing software and computational resources to create many realizations and simulate the classification procedure on the multiple high-resolution models. We must consider the reality of block modeling for the present time. Thus, we are forced to choose an SMU size for the reporting of resources/reserves.

Block estimates may be considered deterministically as is done in the vast majority of kriged block models. The modern probabilistic paradigm is to calculate a probability of waste, probability of ore, and grade of ore for each SMU block by simulation. The probabilities are associated to proportions, for example, 8 out of 10 blocks with an 80% probability of ore are considered as ore, therefore we add 80% of each block's tonnage to the ore tonnes and 20% to the waste tonnes.

Estimates of what the mill will produce and the amount of waste are based on our chosen SMU size. We require a method for selecting an SMU size that yields the same tonnage and grade as conventional grade control practice. This chapter proposes a method that uses conditional simulation to generate multiple realizations of grades, based on which tonnes of ore and grade of ore can be calculated for a range of SMU sizes. Comparison against the results of conventional grade control practice gives the optimum SMU size. Some examples are shown to illustrate the methodology.

4.2 Proposed Methodology

The proposed approach for SMU size selection uses information from both the anticipated grade control practice and realizations from conditional simulation of the long term resource model (or the kriged model if that is the chosen method). The idea is to compare the tonnage and grade obtained from grade control with the tonnage and grade obtained after processing the realizations at a series of different SMU sizes. The first step is to choose a reasonably large and representative production area, A, that likely represents quarterly production. Multiple areas could be chosen and/or different areas could be chosen within different rock types.

The following procedure is undertaken for each representative area, A:

1. Simulate a high-resolution realization accounting for all geological controls, trends, and available data in the region. The resolution of the realization should be 1/3 to 1/10 of the anticipated blasthole spacing and the bench height.

2. Sample the realization with blasthole grades at the anticipated spacing. Dedicated grade control drilling could be considered at this step if that is planned.

3. Simulate the grade control practice to arrive at ore/waste dig limits. The idea is to mimic the actual grade control that will be implemented in the mine. The alternatives include outline-and-average, blasthole kriging, and simulation combined with profit maximization or loss minimization. It is difficult to anticipate all of the operational considerations; however, it may be a good idea to err on the side of conservatism. For example, blasthole kriging is a good starting point for this exercise to be followed by the best simulation and profit maximization. The slight conservatism of the blasthole kriging will be offset by operational dilution.

   The ore/waste dig lines can be further smoothed (some erosion/dilation algorithm) to account for dilution considerations and the fact that they cannot be mined exactly. Then, the ore/waste dig lines are used with the reference high-resolution grades to calculate the expected tonnes of ore, t_o — all tonnes flagged as ore regardless of grade, and the grade of ore, z_o — the average grade of all material flagged as ore. The idea will be to determine the SMU size that matches these reserves.

4. Use the drillhole samples, which are available at a coarser spacing than the blasthole data, to generate multiple realizations of grades at a high resolution. These realizations will match the sparser exploration drilling and the geological controls in an approximate manner. It is important not to be too optimistic, for example, the geological boundaries cannot be frozen for the entire exercise.

5. Choose a range of possible SMU sizes and block average all the realizations to the SMU size by simply calculating a density-weighted average of the grades.

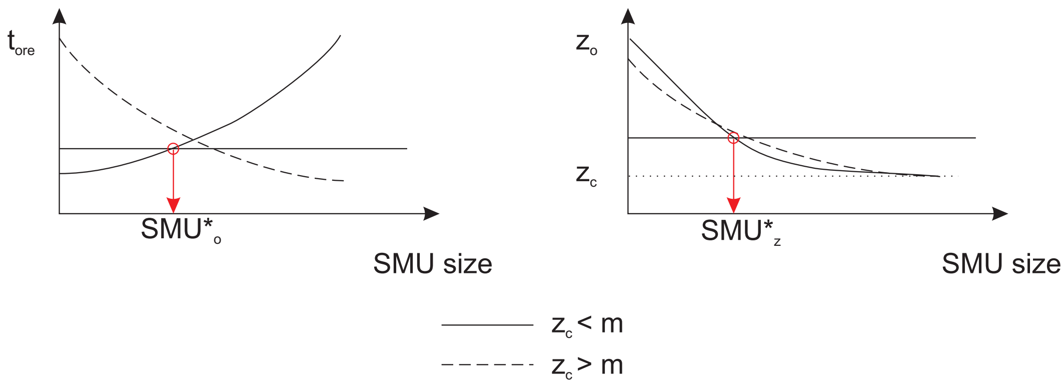

Figure 4.1: Schematic illustration of graphs constructed for optimal SMU size selection: tonnes of ore versus SMU size (left), and grade of ore versus SMU size (right). Note that in each graph the optimal SMU size is selected for the case where the cutoff grade is below the mean grade.

Apply the cutoff grade to all realizations and for all locations, $\mathbf{u} \in A$, and obtain the probability to be above the cutoff grade $(P(Z(\mathbf{u}) \geq z_c))$, probability to be below the cutoff grade $(P(Z(\mathbf{u}) < z_c))$, and the average grade above the cutoff grade $(z_o(\mathbf{u}))$. Calculate the tonnes of ore, t_o, and the grade of ore, z_o.

6. Plot the results of both the grade control and the simulation approach in a two graphs: (1) tonnes of ore versus SMU size, and (2) average grade of ore versus SMU size. In each graph, the conditional simulation results are plotted to yield a functional relationship, while the grade control values provide a single true value that plots as a horizontal line. The optimal SMU size is the size at which these two lines intersect. Figure 4.1 shows a schematic illustration of these graphs, showing two different functional relationships that depend on whether the cutoff grade is above or below the mean grade. The optimal SMU size is shown for the latter case.

Many of the considerations previously mentioned do not intervene directly in this procedure. Moreover, there is a risk that the results are too optimistic because the same geostatistical parameters are used for both the reference realization, grade control and the simulation for resource assessment. The procedure could be refined to account for more factors, and result in a slight increase in the observed SMU size.

4.3 Application

A synthetic example is used to illustrate the methodology. A reference data set is generated via unconditional simulation with a histogram and a variogram. The variogram is arbitrarily chosen with a maximum continuity direction at 35 degrees azimuth with the following model:

$$\gamma(\mathbf{h}) = 0.05 + 0.55 Exp \begin{array}{ccc} a_{hmax} & = & 1100 \\ a_{hmin} & = & 900 \\ a_{vert} & = & 10 \end{array} (\mathbf{h}) + 0.40 Sph \begin{array}{ccc} a_{hmax} & = & \infty \\ a_{hmin} & = & 2000 \\ a_{vert} & = & 10 \end{array} (\mathbf{h})$$

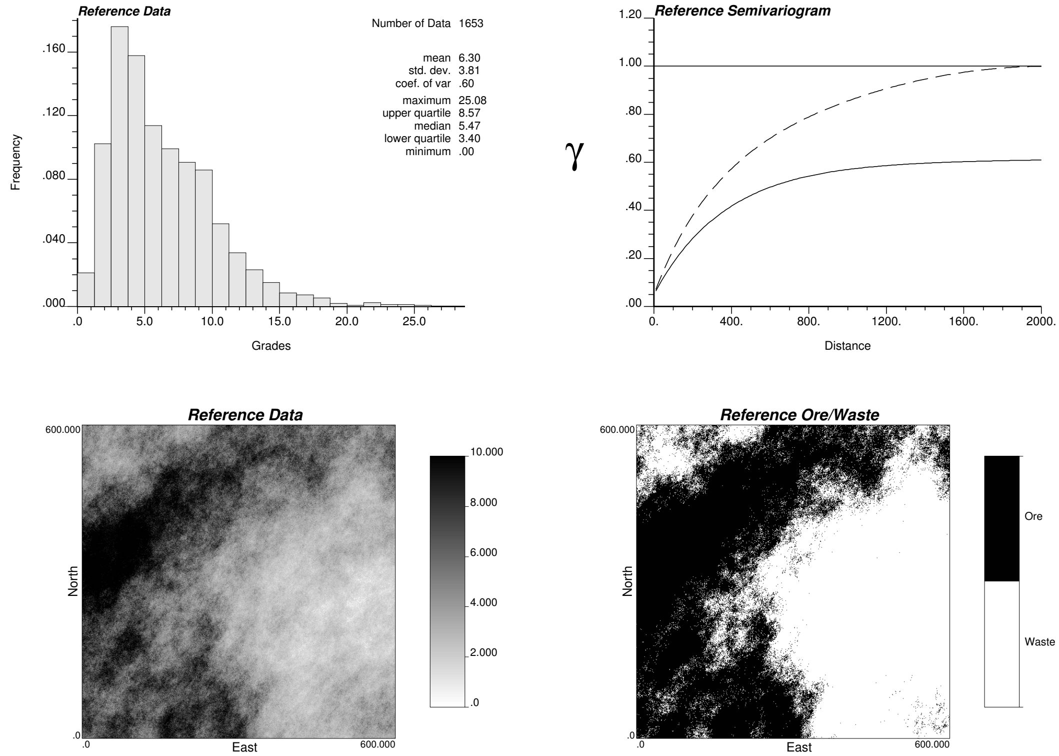

Figure 4.2: Reference model: histogram (top left), variogram (top right), map of reference data (bottom left), and the map of the reference ore-waste classification (bottom right).

The reference data are generated at a resolution of 1m x 1m x 5m that spans an area of 600m x 600m. The block height corresponds to an arbitrarily small 5m bench. Assuming a specific gravity of 2.7, this volume corresponds to a quarterly production volume of just under 5 million tonnes (at nominally 55 000 tonnes/day). A cutoff grade of 5.0% is applied to each location and a reference ore-waste map is obtained. Figure 4.2 shows the reference data histogram, variogram and maps.

Conventional Grade Control. Blasthole data are sampled from this reference map at nominally 10m x 10m spacing. A small random component is added to the sampled data to mimic potential sampling errors in the field. These blasthole data are then used to perform estimation of the grades at a fine 2m x 2m x 5m grid using ordinary kriging (Figure 4.3). For the two maps, the cutoff grade is applied to show only those values strictly above the cutoff grade. An ore-waste contact outline is drawn corresponding to the trimmed map of estimates because it gives a less noisy approximation of the boundary between ore and waste.

This ore-waste outline is applied to the reference grade map to determine the mill's production (Figure 4.4). For an assumed specific gravity of 2.7, there are 2.426 million tonnes of ore at an average grade of 7.35%, and there are 2.434 million tonnes of waste. These become the reference or base values for t_o and z_o for checking against the following simulation approach.

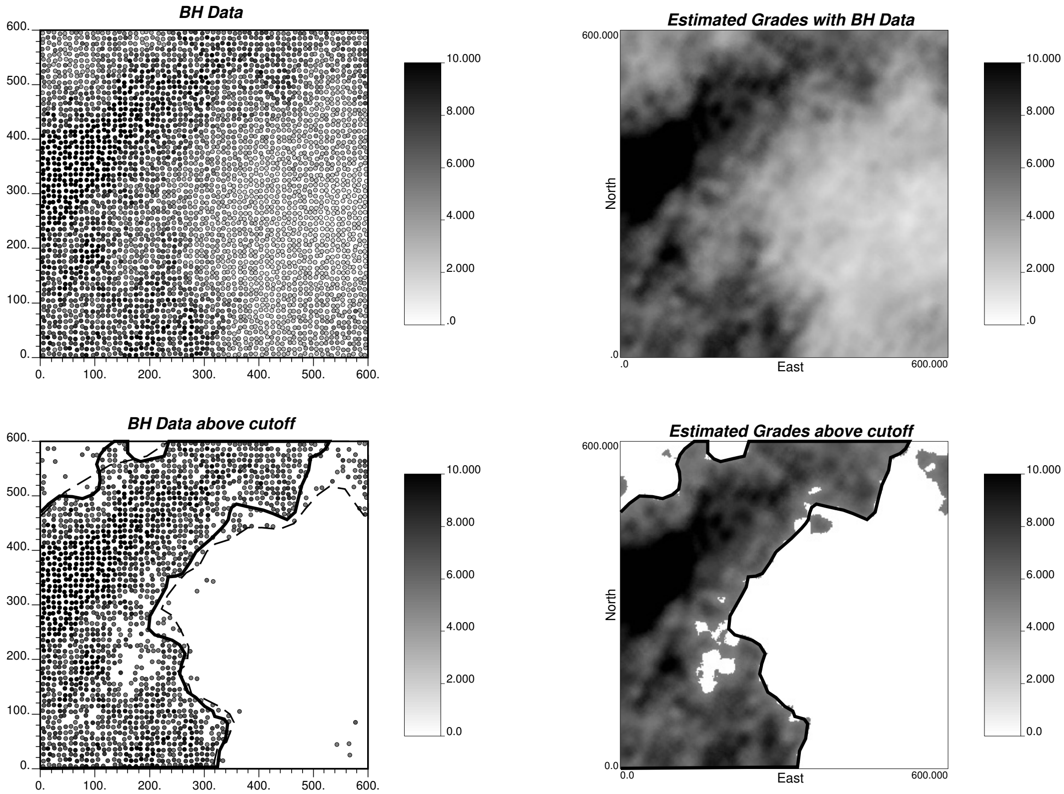

Figure 4.3: Maps of: (1) blasthole (BH) samples at nominally 10m x 10m spacing (top left), (2) estimated grades using ordinary kriging all BH data (top right), (3) only those BH samples above the cutoff grade of 5% (bottom left), and (4) only those estimated grades above the cutoff grade (bottom right). Outline based on the estimated grades map is shown in (3) and (4).

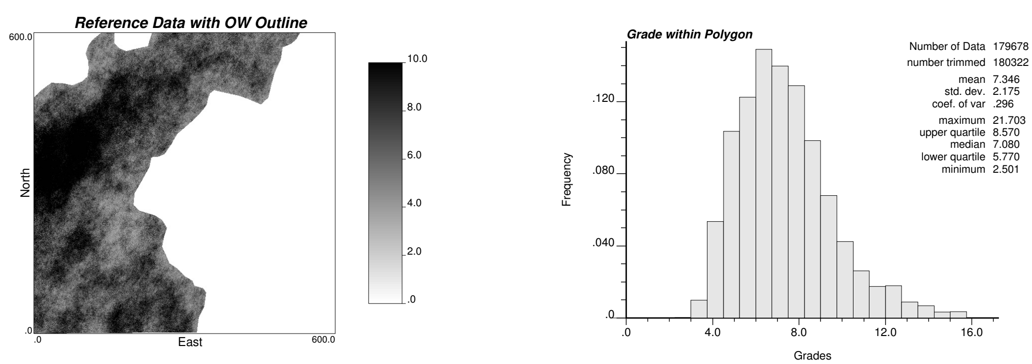

Figure 4.4: Map of reference grade for ore (left) and histogram of grades within the ore polygon (right).

Simulation-based Grade Control. Exploration drillholes are sampled from the reference map at approximately 50m x 50m spacing. These samples are used to construct a conditional simulation model for small 5m x 5m block sizes. The ten realizations generated are then block averaged to a range of possible SMU sizes: 10x10, 15x15, 20x20, 25x25, 30x30, 40x40, and 50x50. Figure 4.5 shows an example of the block averaged results for one realization at the eight different SMU sizes.

For each block model, the cutoff grade is applied and the probability and grade of ore are calculated. Figure 4.6 shows the maps for the probability and average grade above cut off for SMU sizes 10x10, 20x20, 30x30 and 40x40. Using these maps, the tonnes and grade of ore are calculated and compared against the reference values using the ore-waste contacts (Figure 4.7). Based on the tonnes of ore, the optimal SMU size is approximately 20m x 20m; however, based on the grade of ore, the choice of SMU size should be 6m x 6m.

Figure 4.7 shows that for an approximate SMU size of 20m x 20m blocks, the tonnes of ore and waste would match the mill's production. For this same SMU, the grade of ore would be 7.22%, while the mill would produce 7.35% (a percentage difference of 2%). Alternatively, if we tried to match the grade of ore, then the optimal SMU size is approximately 6m x 6m blocks, resulting in an estimated 2.37 million tonnes of ore, instead of the reference production of 2.42 million tonnes of ore (a difference of 4%). For this example, the appropriate choice in SMU size will match the tonnes of ore, since the mismatch between the estimated and actual grade of ore differs by only 2%.

Sensitivity Analysis. To determine the sensitivity of the optimal SMU size to user-selected parameters, the cutoff grade and reference polygon were varied and ore/waste tonnages recalculated for this example.

The cutoff grade was varied from a low of 3% to a high of 7%, and the results of SMU size vs. tonnage/grade were used to calculate an optimal size for each cutoff (Figure 4.8). The optimal SMU size based on grade had inconsistent results that were all very small compared to the SMU size based on tonnage. The optimal SMU size appears to reach a maximum around a 5% cutoff grade. The mean grade for all the ore is 5.211% and the median is 4.991%. It appears the optimal SMU size is largest when the cutoff is near these statistics; however, other cases would have to be examined to confirming this relation.

To analyze sensitivity of the SMU selection to the polygon used for determining the reference values, initial polygons digitized by seven different people were used to obtain different reference values. An semi-automatic dig limit optimization program was also used to find a polygon [13]. Ore tonnages were used to determine the optimal SMU sizes because of the larger spread in the values. The different reference values are shown along with actual recovery at a cutoff grade of 5.0% in Figure 4.8. The resulting optimal SMU sizes range from below 5m to over 50m, suggesting that the results of this kind of study would be better used for comparison rather than obtaining absolute values. Also, the results using different cutoff grades must be done by the same person to give consistent results. Another alternative to obtain consistent results would be to use an optimization software such as the one used here.

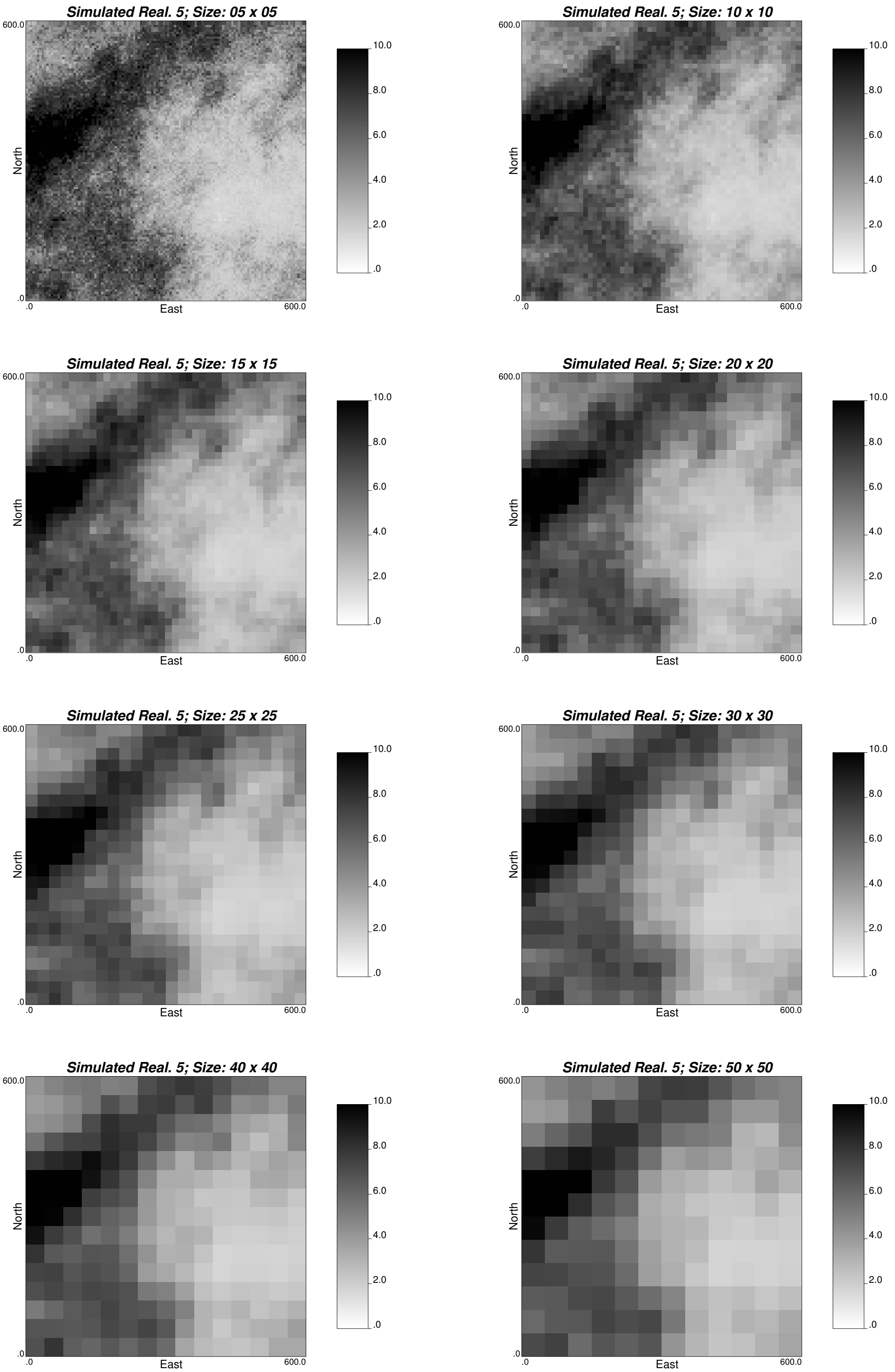

Figure 4.5: One realization from conditional simulation at 5m x 5m blocks (top left). This realization is block averaged to seven different SMU sizes: 10x10, 15x15, 20x20, 25x25, 30x30, 40x40, and 50x50.

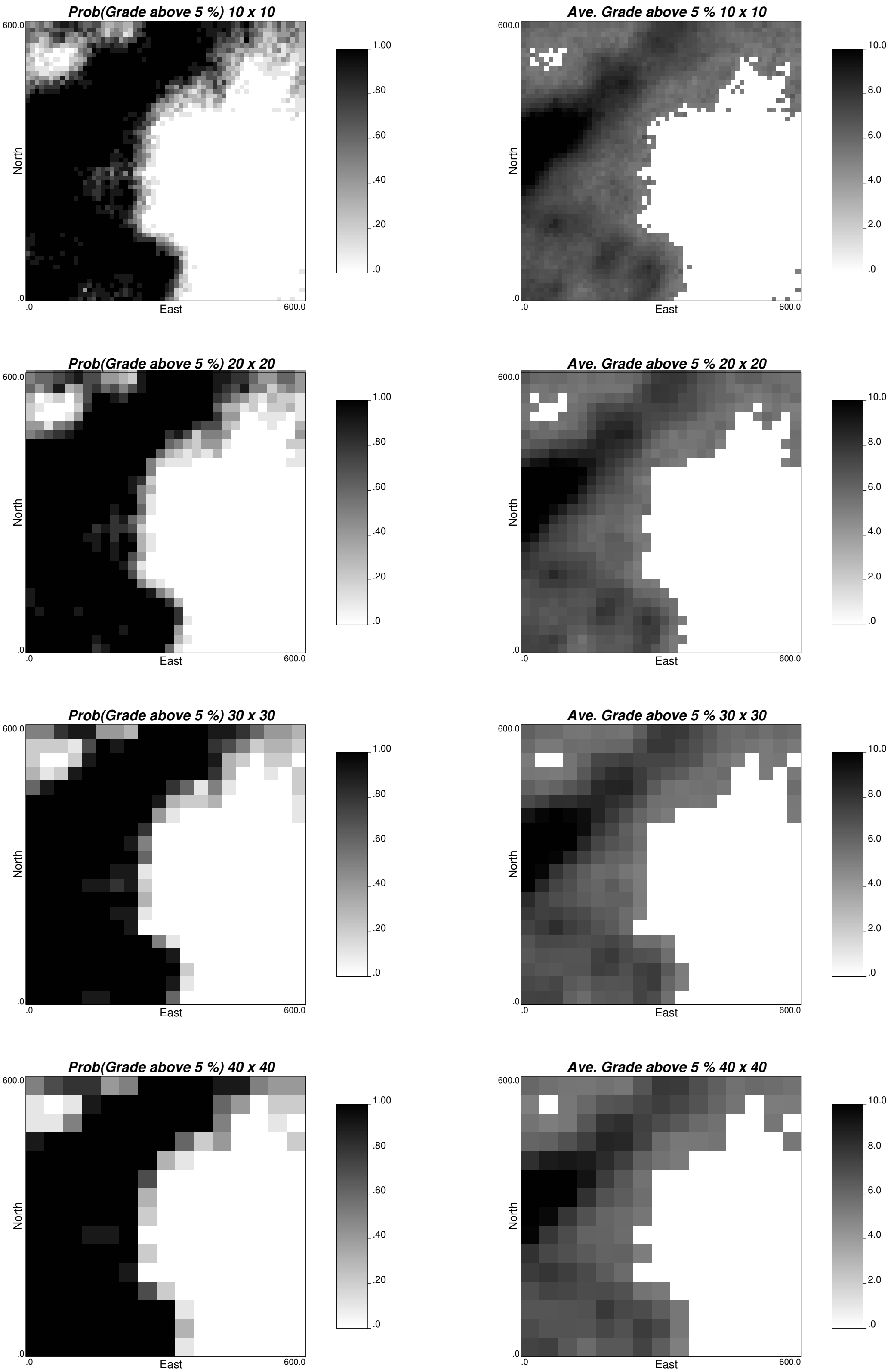

Figure 4.6: Probability (left) and grade (right) of ore maps for SMU sizes: 10x10, 20x20, 30x30, and 40x40.

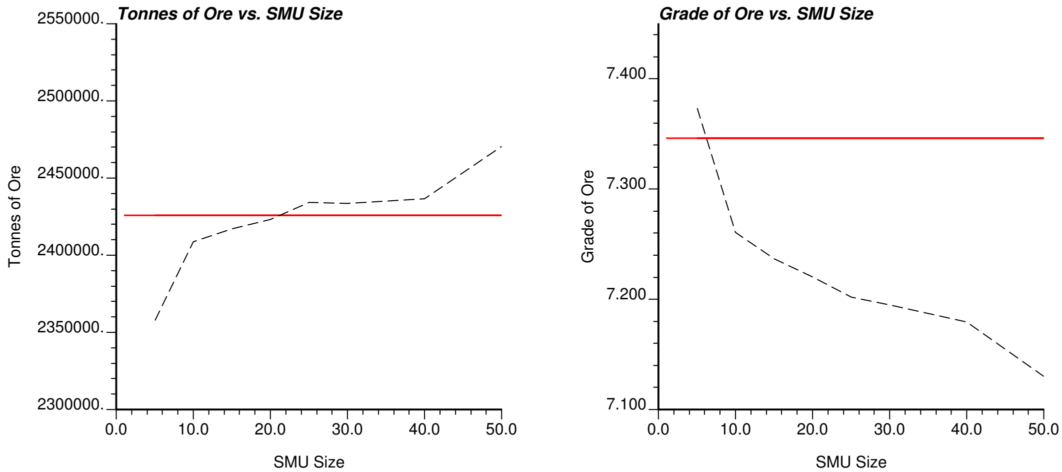

Figure 4.7: Tonnes of ore (left) and grade of ore (right) versus SMU size. The reference values are plotted as solid, horizontal lines.

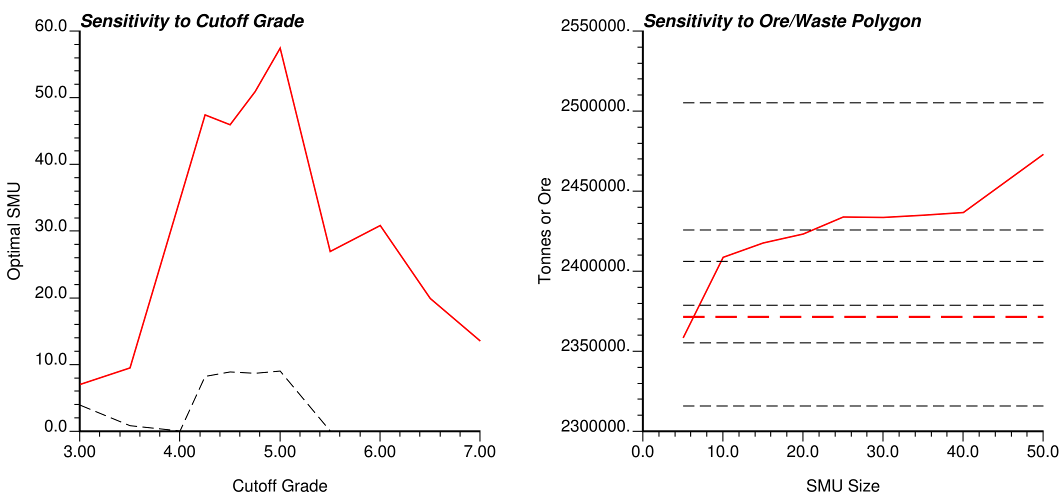

Figure 4.8: Sensitivity of optimal SMU size to cutoff grade (left) and to ore waste polygon used to determine the reference tonnes of ore (right). For the sensitivity to cutoff grade, the solid line corresponds to the optimal SMU based on tonnes, while the dashed line corresponds to the optimal SMU based on grade. For the sensitivity to the polygon, the solid line shows the relation of ore tonnage to the SMU size, while the dashed lines represent the reference tonnes of ore as determined by different ore/waste limits. The thickest, longest dashed line corresponds to the polygon obtained from the semi-automatic dig limit optimization program.

4.4 Discussion

In certain instances, an analytical approach could also be taken to determine the appropriate SMU size. For example, if the distribution of tonnes is known to be normally distributed with mean μ and standard deviation σ^2, then the relation between the recovered tonnes given a cutoff grade can be calculated as ([9], p. 480) (a similar type of function is also available for a lognormal case):

$$T(z) = T_o \left[1 - G\left(\frac{z-\mu}{\sigma}\right)\right] \tag{4.1}$$

Historical production data provides input information regarding the cutoff grade, the recovered and total in situ tonnes, as well as the mean grade of the processed material. Rearranging Equation (4.1), the standard deviation of the ore tonnage is given by:

$$\sigma = \frac{z-\mu}{G^{-1}\left(1 - \frac{T(z)}{T_o}\right)} \tag{4.2}$$

Thus, in order to match tonnes of ore, the appropriate SMU size would have a dispersion variance equal to the square of Equation (4.2), that is $D^2(v, A) = \sigma^2$. We also know that

$$D^2(v, A) = \overline{\gamma(A, A)} - \overline{\gamma(v, v)} \tag{4.3}$$

where

$$\overline{\gamma(A, A)} = \frac{1}{A \cdot A} \int_{A(\mathbf{u})} \int_{A(\mathbf{u}')} \gamma(y - y')dy'$$

and

$$\overline{\gamma(v, v)} = \frac{1}{v \cdot v} \int_{v(\mathbf{u})} \int_{v(\mathbf{u}')} \gamma(y - y')dy'$$

Given the variogram model and the size and geometry of the domain of interest, the calculation of $\overline{\gamma(A, A)}$ can be obtained numerically. If the domain can be considered ergodic, then $\overline{\gamma(A, A)} = \sigma^2_{data}$. The value of $\overline{\gamma(v, v)}$ is given by rearranging Equation (4.3):

$$\overline{\gamma(v, v)} = \overline{\gamma(A, A)} - D^2(v, A)$$

In practice, $\overline{\gamma(v, v)}$ is calculated numerically be discretizing the volume into point locations and the average variogram is calculated over these discretized points. This type of calculation requires knowing the volume over which the variogram must be averaged, which is unknown. For a simple rectangular block with a square base and known height, the sides of the square base can be determined by using the auxiliary functions and charts provided by Journel and Huijbregts ([9], p. 112, 129). Charts are given for the case of a simple, isotropic spherical and exponential variogram. For instance, given that the variogram is spherical with a range of 200m, the bench height is 25m, and $\overline{\gamma(v, v)} = 0.3$, then the ratio of a side to the range (l/a) is approximately 0.38 ([9], p. 129). The size of the SMU is calculated as 0.38*200 or 76m x 76m.

Although charts and auxiliary functions can be used, many simplifying assumptions about the distributional shape, block geometry and even the spatial continuity are required to determine the SMU size analytically. The proposed methodology is more flexible in that simplifying assumptions are not required, and it can be applied to any deposit.

The results of applying the proposed methodology shows that the SMU size does not need to correspond directly with the smallest mineable volume chosen by the mining engineer. Different SMU block sizes account for the fact that dig limits are drawn for grade control so a portion of the field is not mined for ore. The assumption of perfect selection combined with dilution from larger blocks can yield results that match the short term mine practice of selecting only a portion of the field and using samples that are closer together.

The underlying simulation models are generated at a fine scale, consistent with the support of our drillhole samples. These models are always averaged up to larger scales for the purposes of ore reserve estimation. A big advantage of geostatistical simulation is that we can consider multiple SMU sizes (based on this exercise) to match the mills production.

It will not always be possible to match both the tonnes of ore and the grade of ore that would be produced by the mill. The graphs obtained from the example shows that the optimal SMU size depends on whether we are matching tonnes or grade. The tonnes of waste is also an issue, but this is inversely related to the tonnes of ore so consideration of tonnes of ore automatically accounts for the tonnes of waste.

Selection of the appropriate SMU is a compromise between getting the right estimates for tonnage of ore and getting the right grade for this material. This compromise depends on the magnitude of difference associated with selecting one SMU over the other, which amounts to determining (1) whether the discrepancy between the actual and estimated grade is too significant to accept even though the tonnage is correct, or (2) whether the discrepancy between the actual and estimated tonnage is too large to accept even though the grade of ore is correct.

It is possible to obtain a graph where the simulation results at different SMU sizes intersects the reference value at multiple sizes. Figure 4.9 shows an example of one such case. More studies are required to sort all of these details out. Our goal could be to match metal content. Another concern is that the simulation results may never intersect the reference value (Figure 4.10). This is related to the available data — there may be too few samples to obtain "good" simulations. The distribution of sampled drillhole data may be inconsistent with the distribution of more closely spaced blasthole grades. Considering multiple realizations and/or discarding realizations that are too inconsistent would make the results more stable.

Although ore/waste limits may be drawn by the mine geologist, some dilution of the delineated zone is expected due to site operations such as errors in survey staking, and sloughing of blasted material. These result in dilution due to mine operations. There are a number of ways that we could account for this type of dilution. The initial ore/waste limits identified in conventional grade control practice can be dilated and eroded to account for these forms of operational dilution. The tonnes and grade of ore can then be calculated using these modified ore/waste outlines. Comparison with the simulation results would then be checked using these revised estimates. The selection may also be improved by on-site visual refinements by the geologist or equipment operator. This is more difficult to account for in a simulation context.

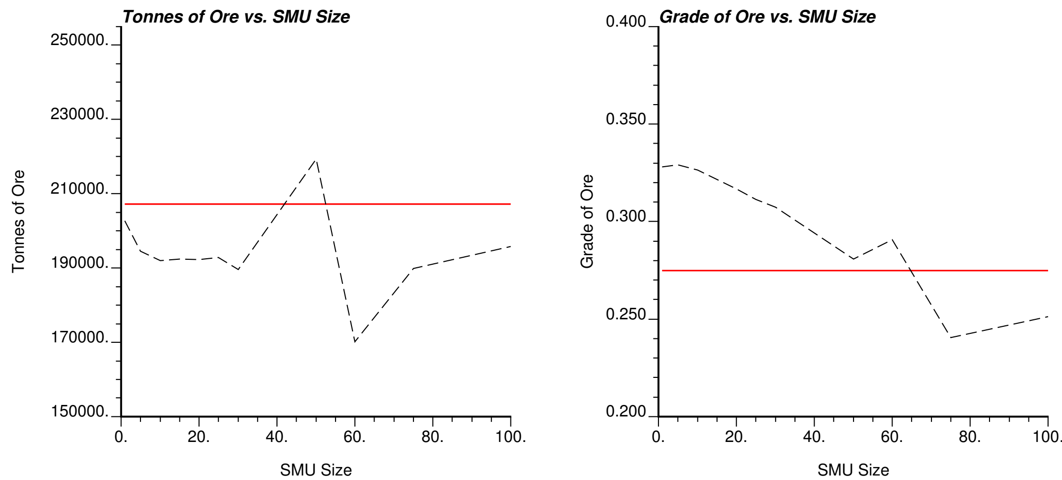

Figure 4.9: Example of multiple optimal SMU sizes from graphs of tonnes of ore (left) and grade of ore (right) versus SMU size. The reference values are plotted as solid, horizontal lines.

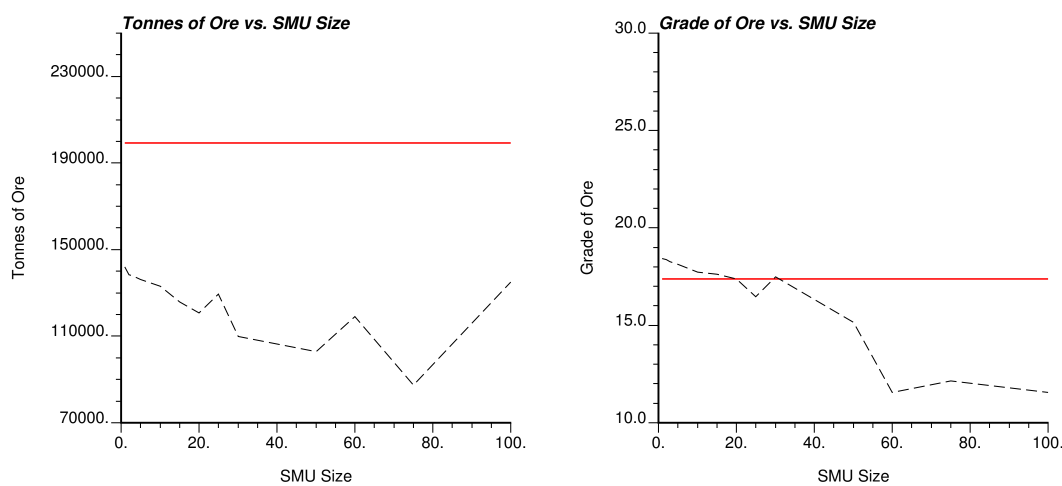

Figure 4.10: Example of non-intersection of graph of tonnes of ore (left), and multiple intersection of grade of ore (right) versus SMU size. The reference values are plotted as solid, horizontal lines.

External dilution due to sharp geological boundaries is another important consideration. These contacts may distinguish between the mineralized host rock and barren rock; thus, poorly delineated ore/waste contacts can result in a significant amount of dilution. In these cases, more exploration drilling may be required to identify these regions. Simulation of two different populations is recommended to obtain more reliable reserve estimates and, consequently, better SMU size selection.

In addition to SMU size, sample quality and spacing impact recoverable reserves. As the sample spacing increases, or as the sample error goes up, there will be a larger proportion of blocks that are misclassified. These misclassified blocks are ore blocks mined as waste and waste blocks mined as ore. An exercise is presented in the following Chapter to quantify the effect that sample spacing and error have on the recovered reserves. The effect can be included into the reserve estimation process by adjusting the SMU size accordingly.

Chapter 5

Information Effect

Ore and waste material are classified with incomplete and imperfect information. The blasthole or grade control drilling is spaced too far apart for error free estimates of grade. This leads to some ore being classified as waste and some waste being classified as ore. This incomplete knowledge and the resulting misclassification is called the information effect [17].

The information effect influences the recoverable reserves at the time of mining. It should also be accounted for in the early stages of reserve estimation. This chapter presents a methodology for calculating the information effect and the impact that it has on the estimated reserves. The results from the proposed methodology are compared to a theoretical result.

This chapter proposes a methodology for calculating the information effect and accounting for it in the subsequent reserve calculation. This involves generating a reference model, outlining the methodology, performing the calculations and comparing the results.

5.1 Methodology

There are two methods that can be used for calculating the information effect: (1) using an assumption about the information effect and incorporating it into a change of support model, or (2) simulating the expected grade control sampling and calculating the effect on the reserves directly. The first method accounts for the information effect by smoothing the block scale distribution. The variance of the block distribution is reduced to account for the information effect. For a low cutoff, the number of tonnes recovered increases at a diluted ore grade. At a high cutoff, the number of tonnes will decrease with a lower ore grade. The second method uses a reference model and simulates sampling classification and the affect on reserves.

The theoretical approach requires an assumption for the impact that the information effect has on the recovered ore. The larger the impact, the smoother the block scale distribution becomes. For example: say the variance of blocks v is estimated to be 20.0 using a gammabar value. Reserves could be calculated using the calculated block variance, but that assumes that the selection of the blocks v is perfect, which is never the case. To account for the misclassified blocks, the variance can be reduced and the reserves calculated with the reduced variance [2]. It is unclear how to reduce the block variance. It is usually done by using experience at similar deposits. The simulation approach can be used to calculate the variance reduction that should be used for a specific sampling regime.

The simulation approach aims at quantifying the information effect by simulating the grade control drilling and material classification. There are four steps in simulating the grade control and mining: (1) simulate the grade control drilling by extracting samples from the reference model, (2) use the extracted samples to estimate the gold grade for the mining blocks, (3) classify the grade control model using the kriged estimates, and (4) calculated the profit for the area using the reference model and classification model. No explicit assumptions are made for dilution. We assume that each block can be selected freely from the blocks around it.

A mining block size of 5m was used for this example. The reference model will be upscaled to provide an ideal profit that is only attainable with perfect information and selection. The results of the distribution upscaling and information effect will be compared to the ideal profit.

Profit will be used to compare the results. Grade, tonnage, or quantity of metal could have been used, but profit summarizes all of these variables with a single number. The profit calculation is straightforward: the profit for each block is calculated using the reference model for the block grades and the classification model to decide if it is ore or waste. The profit for each block is calculated as:

$$profit\left(\mathbf{u}\right)=\begin{cases} z_v\left(\mathbf{u}\right)\cdot p\cdot rec-c_o-c_t, & if\ z_v^*\left(\mathbf{u}\right)\geq z_c \\ c_w, & otherwise \end{cases}$$

where $z_v^*\left(\mathbf{u}\right)$ is the estimated grade at location $\mathbf{u}$, $z_v\left(\mathbf{u}\right)$ is the actual grade from the upscaled reference model, z_c is the cutoff grade, p is the gold price, rec is the processing recovery, c_o is the cost of mining ore, c_t is the cost of processing ore, and c_w is the cost of mining waste. The total profit is the sum of the profit for each block in the model. The following parameters were used for the profit calculation:

$$c_t = 12\ ^{\$}\!/_{t}$$

$$c_o = 2\ ^{\$}\!/_{t}$$

$$c_w = 1\ ^{\$}\!/_{t}$$

$$p = 18\ ^{\$}\!/_{g}$$

$$rec = 80\%$$

The cutoff grade is a function of the above economic parameters. The cutoff grade was calculated using the following formula:

$$\begin{aligned} z_c &= \frac{c_t+(c_o-c_w)}{p\cdot r} \\ &= 0.90\ ^{g}\!/_{t} \end{aligned}$$

The calculated cutoff grade is within an acceptable range for open pit gold mines.

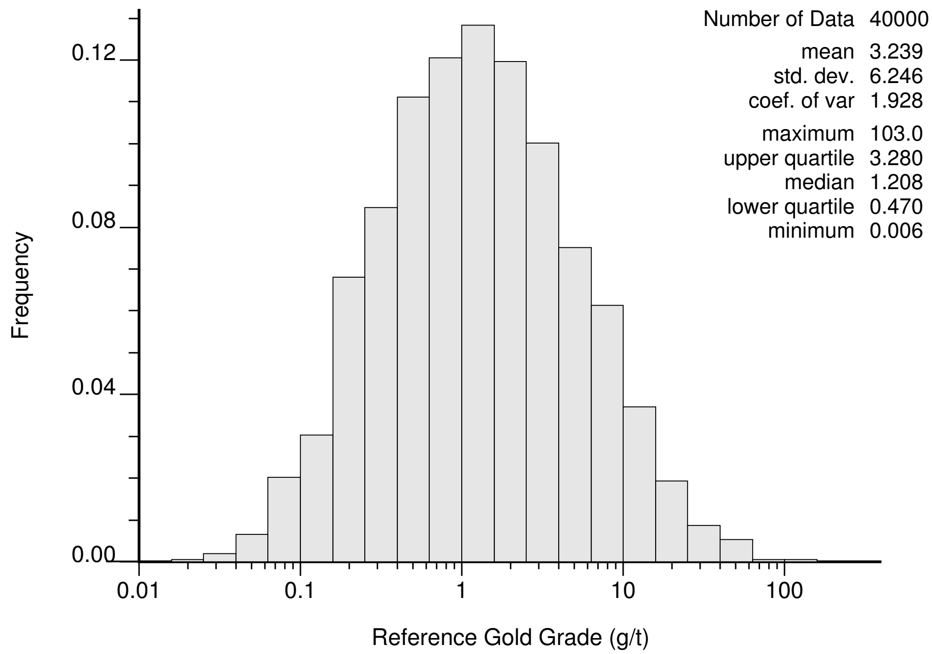

Figure 5.1: Reference gold distribution.

5.2 Reference Distribution and Model

A reference gold grade model was constructed using unconditional simulation and a reference gold distribution. The reference model is shown in Figure 5.1. The following variogram was used to construct an unconditional model using simulation:

$$\gamma(\mathbf{h}) = 0.1 + 0.9 \cdot Sph_{\substack{ahmax=40\\ahmin=40\\avert=10}}(\mathbf{h})$$

Back transforming the unconditional simulation provided the 2-D reference gold model; see Figure 5.2. The reference model is 200 blocks x 200 blocks and each block is 1m x 1m x 5m. The reference model will be used for simulating grade control sampling, classification and calculating the actual recovered ore.

The ideal profit was calculated by upscaling the reference model to the 5m x 5m mining block. The ideal profit will never be attained during mining. It was only used as a reference for comparing results. The maximum profit attainable is 16 million dollars.

5.3 Theoretical Results

The profit for SMU blocks of different sizes was calculated using the economic parameters and the upscaled block distribution from the discrete Gaussian model. To upscale the reference distribution to larger blocks, the block scale variance is needed. The block variance is calculated using gammabar values for the different block sizes. Recall the dispersion variance equation:

$$\begin{aligned} D^2(v, A) &= D^2(\cdot, A) - D^2(\cdot, v) \\ &= \sigma^2 - \bar{\gamma}(v, v) \end{aligned}$$

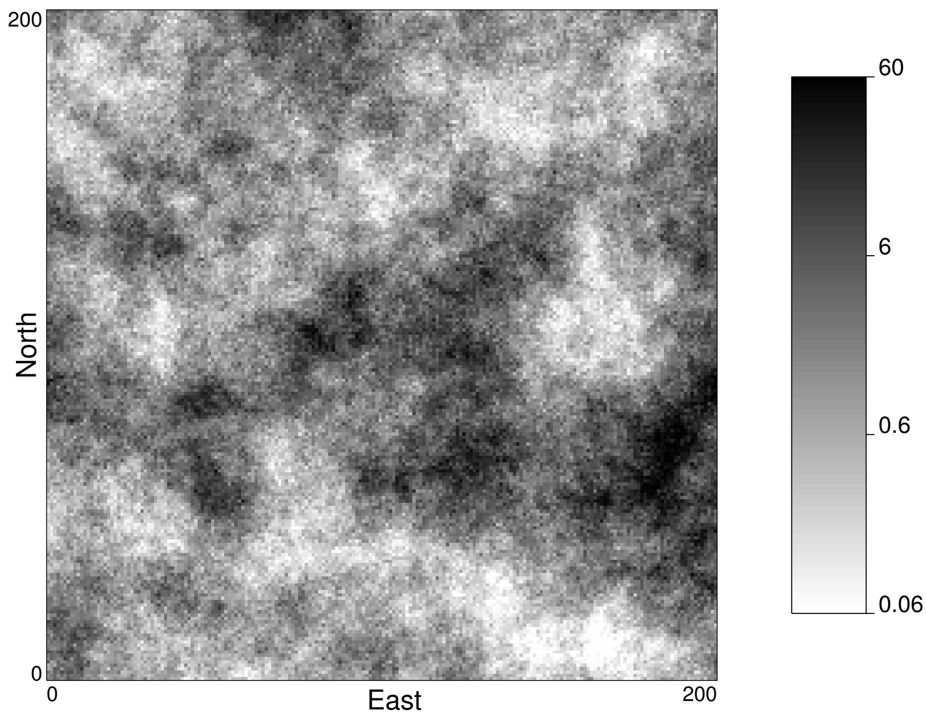

Figure 5.2: Reference 2-D gold model.

Figure 5.3 shows the block variance for the different SMU sized blocks. The variance of the blocks decreases as the block size increases; this was expected. The reference distribution was upscaled to the larger block supports using the DGM.

As the variance of the SMU decreases the distribution becomes smoother. This is analogous to introducing dilution to the recovered ore. Dilution decreases the recovered grade and will often reduce the profit seen from the model area. The profit for the different SMUs is shown in Figure 5.4. As expected, the profit decreases as the SMU size increases.

For a 5x5m SMU, the profit from the discrete Gaussian model matches the ideal profit exactly. It is unrealistic to expect that the ideal profit is achievable. The imperfect information used to classify material at the time of mining will result in a lower profit. A larger effective SMU size is needed for estimating the recoverable reserves with the DGM. The simulation study presented next can be used to choose the larger effective SMU for reserve estimation.

5.4 Simulation Results

The ore waste classification improves as the amount of grade control sampling increases. Sparse grade control drilling results in more misclassified material. Simulated samples were taken at a drillhole spacing starting at 1m x 1m and increasing up to 35m x 35m. Kriging was done for each case to get a classification map and then a profit was calculated for each sample spacing case.

Consider the case where the grade control samples are 12m apart. The simulated samples are shown in Figure 5.5. The samples were extracted on a regular grid from

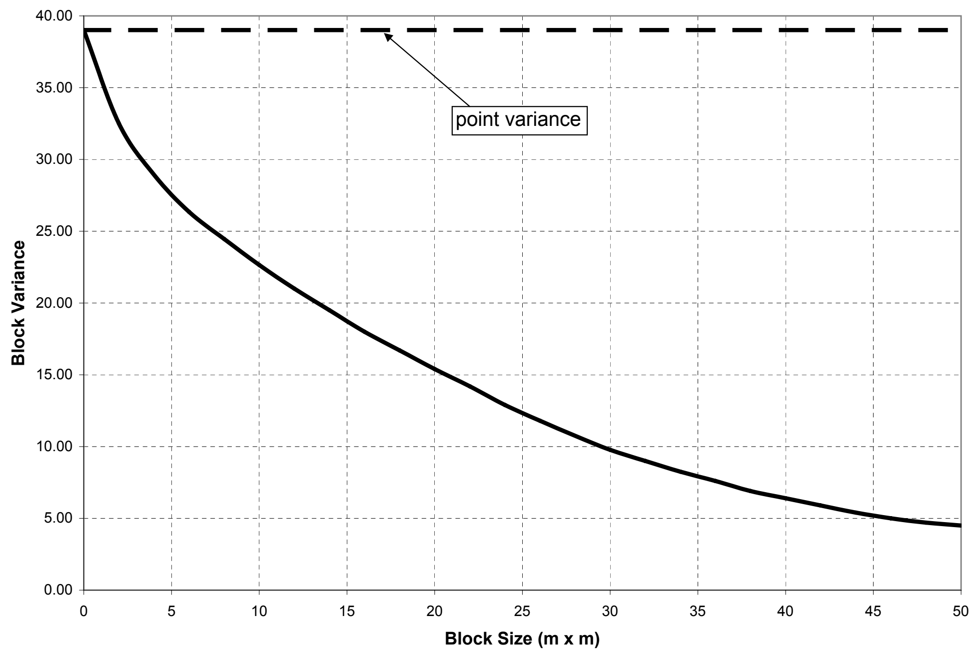

Figure 5.3: SMU Variance versus SMU size.

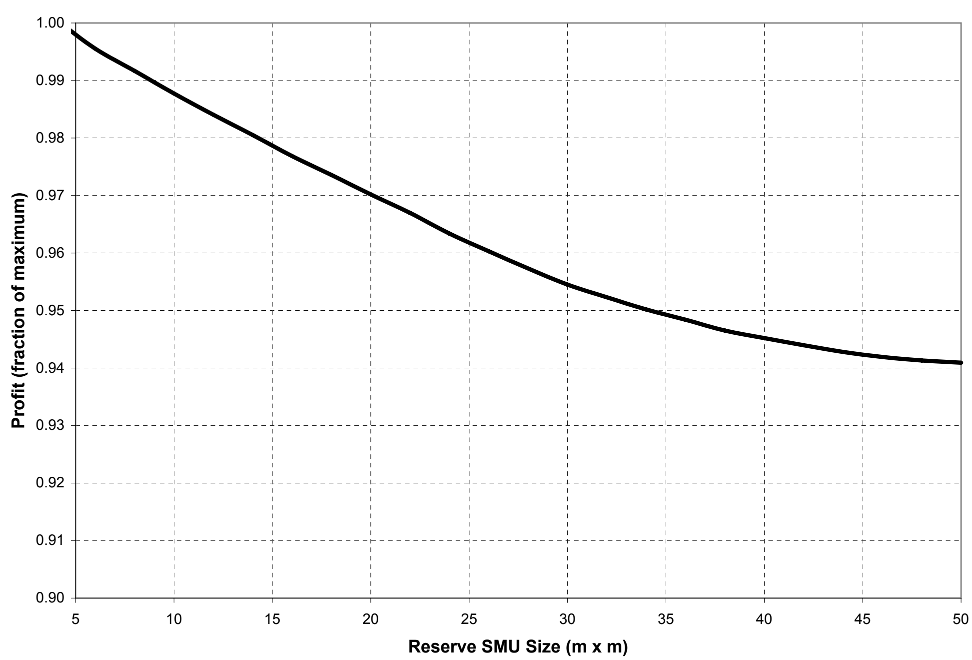

Figure 5.4: Profit from the Discrete Gaussian model versus SMU size.

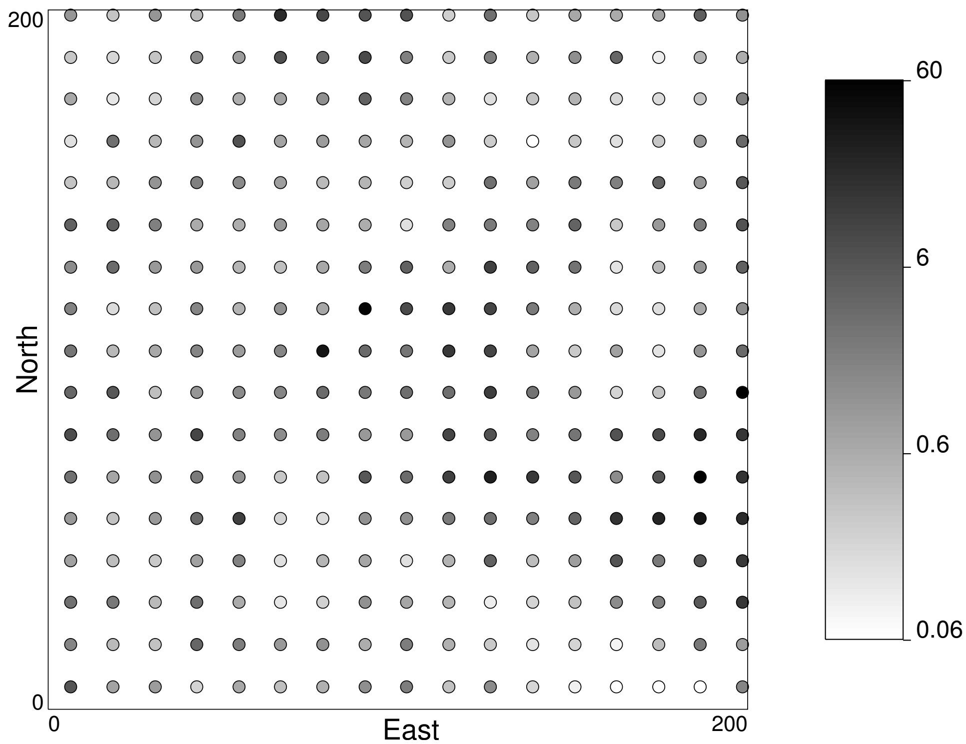

Figure 5.5: Simulated grade control samples.

the reference model and then used to estimate the grade of the SMU blocks. An ore/waste indicator was calculated from the kriged grade control model. Where the estimate was above the cutoff of 0.9, the material was considered ore, and where the grade was less than the cutoff the material was considered waste. The ore/waste map is shown in Figure 5.6. The profit was calculated using the ore/waste classification and the reference model. With a sample spacing of 12m, the profit was 15.6 million dollars. This is 97.5% of the ideal profit. These steps were repeated for the different sample spacing scenario's considered. Part of that 2.5% would be required for mineability.

Figure 5.7 shows the profit versus sample spacing. When the sample spacing is 1m x 1m, the classification model from kriging matches the ideal classification model exactly because every value is sampled. As the sample spacing increases the profit realized decreases. If the proposed grade control sampling is known, the percent decrease from the ideal profit is known. The percentage reduction can be used to determine the effective SMU size that can be used for reserve estimation.

5.5 Comparison

Both the DGM and simulation based approaches for quantifying the information effect produced the predicted results. For the theoretical case, as the block variance decreases, the block scale distribution becomes smoother, or less selective, and the profit realized decreases. And for the simulation case, as the sample spacing increases the quality of the classification decreases and the resulting profit decreases as well.

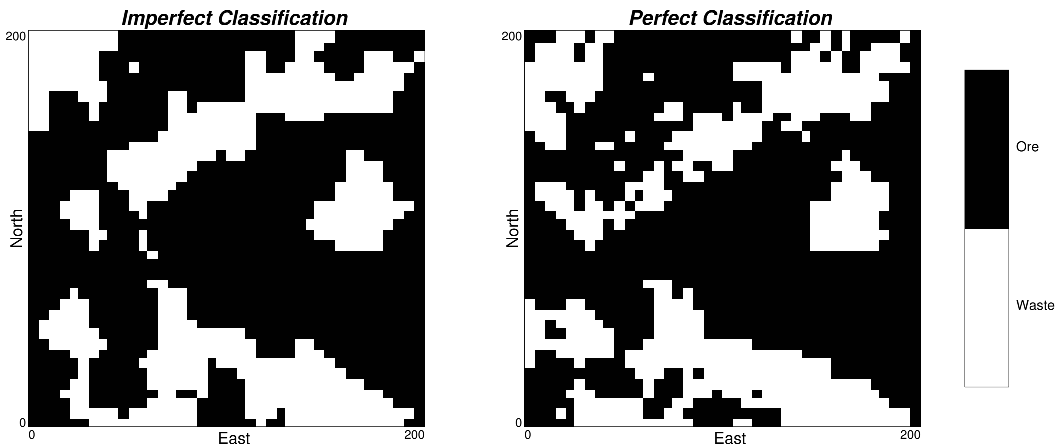

Figure 5.6: Comparison of imperfect and perfect ore/waste classification. The imperfect classification is on the left and the perfect classification is on the right.

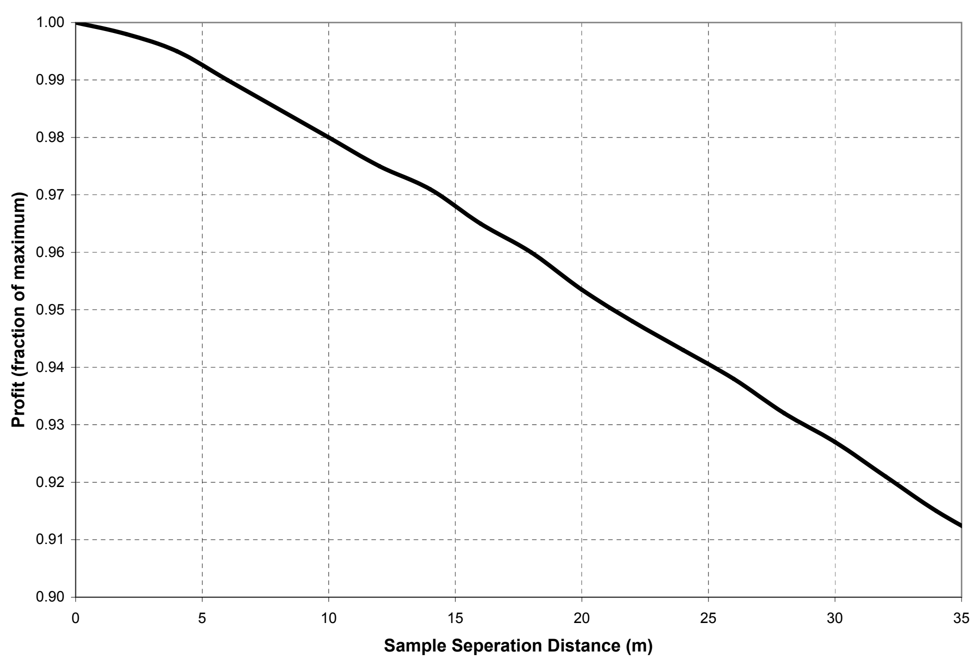

Figure 5.7: Profit from the simulation approach versus sample spacing.

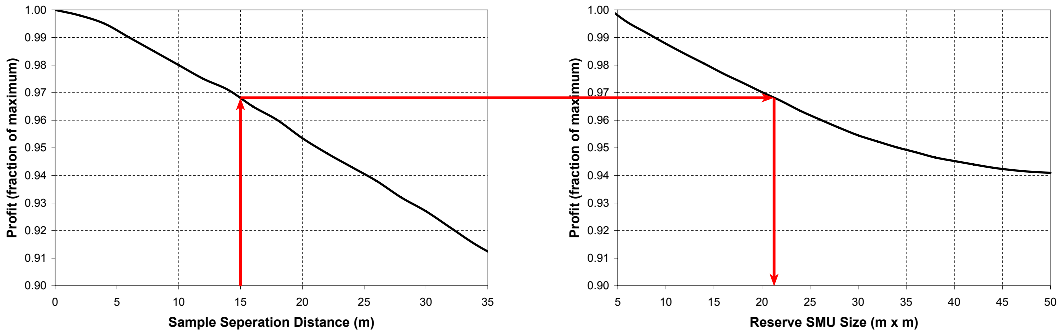

Figure 5.8: Choosing an SMU size.

The simulated sampling, grade control, and mining results can be combined with the theoretical change of support results to choose an SMU size that can be used for reserve estimation. Choosing the reserve SMU size in this manner allows for the information effect to be incorporated into reserves calculated from a change of support model or from a simulated model.

Recall the profit versus sample spacing plot in Figure 5.7 and the profit versus SMU size in Figure 5.4. Say that the mine is planning to drill grade control samples on a 15x15m spacing and the mining equipment is capable of mining 5x5m blocks. According to the results of the study, the profit using a 15x15m grade control sampling program will be just under 97% of the total attainable profit. To accurately estimate the mineable reserves, while accounting for the information effect, an SMU size that will estimate the same profit as the grade control study should be used. The SMU size gives the same estimated profit is 21m. Figure 5.8 shows how the SMU size was chosen for this example. Reserves estimated with an SMU size of 21m will account for the information effect.

Chapter 6

Conclusions and Future Work

Recoverable reserves are the backbone of any mining project. Mining companies will invest millions of dollars to develop a mine using an estimate of the recoverable reserves. Over or under estimating the reserves can result in a significant loss to the mining company. There are different methods for calculating recoverable reserves and several important considerations that must be taken into account. The goal of any recoverable reserve estimate is to predict the tonnes and grade of the material that will be mined as ore once mining commences. The estimate must account for the mining equipment, the imperfect information that will be available at the time of mining, dilution and other operational considerations.

Selective mining units (SMUs) are used for calculating reserves. The notion of a selective mining unit is confusing. Many people associate an SMU with the mining equipment. This is partially correct; however, for reserve estimation, the SMU does not coincide with the mining equipment selectivity, it has to account for the mining equipment, the imperfect information at the time of mining and dilution. The SMU size may change from initial mine exploration and planning to the mine development and mining stages.

Mining equipment cannot perfectly select ore and waste. Inevitably, some ore will be mined as waste and some waste will be mined as ore. The size of the equipment and the nature of the deposit contribute to dilution. Assumptions regarding selectivity and dilution are made during reserve estimation. These assumptions come from previous experience on similar deposits. Dilution can be accounted for by adjusting the SMU size. Increasing the SMU size for calculating reserves incorporates dilution into the reserve estimate. Chapter 4 presented a methodology for selecting the SMU size to account for mining dilution.

The classification of ore and waste for mining will always be done with limited information. The information effect has a similar effect on the reserves as dilution; ore lost as waste and waste included as ore. The spacing of the grade control or blasthole samples relates to the information effect. Dense drilling provides more information about the deposit and improved selection, while widely spaced sampling leads to a large information effect, that is, worse selection. The information effect can be included in a reserve estimate by increasing the size of the SMU. As the SMU size increases, it implies an increasing information effect, or that an increasing number of blocks are being misclassified. Chapter 5 presented a methodology for selecting the SMU size to account for mining dilution.

The SMU size should account for the information effect and dilution simultaneously. Although the methods were presented in different chapters, they could be applied at the same time so that one SMU size for reserve calculation can be determined. Reserves calculated with this SMU should provide an accurate estimate of the recovered reserves.

Data based methods use the sample data and a change of support model to estimate the upscaled SMU distribution. Recoverable reserves are then calculated using the upscaled histogram. They provide no spatial location of the high and low grade areas. Therefore, they cannot be used for mine planning or pit optimization. It has been shown that global reserve estimation methods provide accurate estimates of the recoverable reserves. For this reason, they are typically used to tune the local reserve estimates.

The affine change of support model makes an unreasonable assumption that the histogram will not change shape as the support volume increases. For this reason, the affine model is not recommended. The indirect lognormal model assumes that the point scale distribution and the block scale distribution are lognormal. The shape of the histogram does change as the support volume increases for other distributions. The indirect lognormal model will produce acceptable results when the data are approximately lognormal. The discrete Gaussian model works on the normal score transform of the data and a fitted polynomial to perform the change of support. The discrete Gaussian model can be fit to almost any input distribution, but it makes the assumption that the distribution tends to Gaussian as the block size increases.

Local reserves are calculated with estimation or simulation. These include building a block model and assigning a grade to each block in the model. Estimation methods provide a single grade estimate at each location, while simulation can provide the uncertainty at a particular location.

Simple and ordinary kriging are common methods for estimating local reserves. Ordinary kriging does not rely on a global mean. The estimates from simple and ordinary kriging are smooth. They do not accurately represent the variability of the deposit and do not account for the future information that will become available. In general, they do not provide good estimates of the recoverable reserves.

Although kriging does not accurately predict the reserves, a block model is needed for mine planning and pit optimization. Reserve tuning and estimate correction methods have been developed to correct kriged estimates so they match global reserves. The kriging parameters could be adjusted to make the kriging reserves match the reserves calculated from the discrete Gaussian model. A correction has been proposed for simple kriging that corrects the smoothing effect. It aims to increase the variability of the kriging estimates so that they match the SMU variance. This has the effect of making the distribution of estimates the same as the distribution of the SMUs from a change of support model. Both of these corrections aim at forcing the reserves from a local estimate to accurately predict the global reserves.

Kriging can be used to estimate blocks larger than the SMU with confidence. The smoothing effect of kriging can be mitigated by estimating blocks that are large in relation to the data spacing. Ordinary kriging of larger panels combined with uniform conditioning produced good results. Uniform conditioning estimates the distribution of the smaller SMU blocks within the larger panels using the discrete

Gaussian model for the change of support. The panel estimates are more robust than the smaller SMU estimates and uniform conditioning correctly estimated the SMU distribution within each panel. A drawback of uniform conditioning is that it does not provide a spatial location of the SMU sized blocks. This makes it difficult to do a detailed mine plan or pit optimization compared to a block model at the SMU scale.

Simulation methods are useful for constructing local reserve models for the purpose of uncertainty assessment. The benefit of simulation is the ability to assess local uncertainty. The local uncertainty could be for a single block, a month of a mine plan, a year of production, or for the life of the mine. Simulation is the only practical way to assess the uncertainty for an arbitrary production period. The downside of simulation is that it takes more time to generate and post process the realizations compared to the histogram based methods or uniform conditioning.

Averaging, or the expected value, is useful for presenting a summary of a simulation study. The expected value must be taken after any non-linear transformation or transfer functions. For recoverable reserves, that means the expected value of the realizations should not be calculated before the grade tonnage curve, the expected value should be taken from the multiple grade tonnage curves calculated from the multiple realizations.

Summary of Contributions

This thesis has reviewed a variety of techniques for recoverable reserve estimation. Perhaps the most important contribution is this comprehensive review in light of recent developments in geostatistical simulation. Aspects of these would form the basis for establishing best practice standards for reserves assessment. More specifically, the following contributions should be considered:

- **SMU size calibration:** Selecting an appropriate SMU size requires consideration of the mining equipment, bench height, blasthole sampling, grade control practice, and affect of dilution. Chapter 4 presented a methodology to determine the optimal SMU size to match actual production. Actual production is simulated on a representative area by simulating the collection of blasthole data and the consequent grade control. Then, the geostatistical resource estimation procedure was implemented for a range of SMU block sizes and the SMU size that gives a reasonable match to the actual production is recommended. The optimal SMU size will yield an estimate of tonnes of ore and grade of ore that is close to the actual production.

- **Information effect:** Ore and waste are mined with incomplete information. As the sample spacing increases, there will be a larger proportion of blocks that are misclassified. Chapter 5 presented a methodology for determining an effective SMU size that accounts for the information effect. Grade control sampling, ore/waste classification, and profit were simulated from a reference model for a range of sample spacing. The profit from the simulation exercise was then compared to the profit from a theoretical change of support model using a range of SMU sizes. An effective SMU size can be chosen so that estimated reserves match the reserves calculated taking into account the information effect.

Notwithstanding these contributions, there are many areas for future research.

Future Work

Reserve estimation requires an SMU. Choosing the SMU size is a difficult task. Even though some methodologies were developed in this thesis, more work is needed to understand the mining process and how to account for it in the reserve estimation. An area for future work is to build a mine simulation program that will simulate the shovel mining, loading the trucks, stockpiling and feeding to the plant. The simulator could be used to assess the impact of a grade control sampling program, the effect of different sized equipment, and other significant variables.

More tools are needed for the assessment of uncertainty. Many advancements have been made over the past few years for building complex multivariate simulated models, but little progress has been made for post processing and using the results to their full potential. A simulated model could be used to assess the uncertainty in the first 5 years of a mine plan. If the uncertainty is too large, the areas of high uncertainty could be targeted for infill drilling to reduce the uncertainty to an acceptable level.

Significant effort is required to build simulated realizations to assess uncertainty. Carrying the uncertainty through all stages of mine planning would be a benefit to mining companies. There are currently no mine planning tools that can process multiple realizations at one time. A true optimized pit needs to account for all possible grade realizations, not just a single realization.

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